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Juan P. Sesmero

University of Nebraska, Lincoln, juampase@hotmail.com

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**ESSAYS ON EQUITY-EFFICIENCY TRADE OFFS IN ENERGY
AND CLIMATE POLICIES**

By

Juan P. Sesmero

A DISSERTATION

Presented to the Faculty of

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ESSAYS ON EQUITY-EFFICIENCY TRADE OFFS IN ENERGY AND CLIMATE POLICIES

Juan P. Sesmero, Ph.D.

University of Nebraska, 2010

Adviser: Lilyan E. Fulginiti

Economic efficiency and societal equity are two important goals of public policy. Energy and climate policies have the potential to affect both. Efficiency is increased by substituting low-carbon energy for fossil energy (mitigating an externality) while equity is served if such substitution enhances consumption opportunities of unfavored groups (low income households or future generations). However policies that are effective in reducing pollution may not be so effective in redistributing consumption and vice-versa. This dissertation explores potential trade-offs between equity and efficiency arising in energy and climate policies.

Chapter 1 yields two important results. First, while effective in reducing pollution, energy efficiency policies may fall short in protecting future generations from resource depletion. Second, deployment of technologies that increase the ease with which capital can substitute for energy may enhance the ability of societies to sustain consumption and achieve intertemporal equity.

Results in Chapter 1 imply that technologies more intensive in capital and materials and less intensive in carbon such as corn ethanol may be effective in enhancing intertemporal equity. However the effectiveness of corn ethanol (relative to other

technologies) in reducing emissions will depend upon the environmental performance of the industry. Chapter 2 measures environmental efficiency of ethanol plants, identifies ways to enhance performance, and calculates the cost of such improvements based on a survey of ethanol plants in the US. Results show that plants may be able to increase profits and reduce emissions *simultaneously* rendering the ethanol industry more effective in tackling efficiency.

Finally while cap and trade proposals are designed to correcting a market failure by reducing pollution, allocation of emission allowances may affect income distribution and, hence, intra-temporal equity. Chapter 3 proves that under plausible conditions on preferences and technology increasing efficiency requires greater transfers to low income households the higher the effect of these transfers on the price of permits and the lower their effect on the price of consumption goods. This denotes market conditions under which efficiency and equity are complementary goals.

To my wife

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CHAPTER 1

ENERGY POLICY AND SUSTAINABLE CONSUMPTION

1. Introduction

Whenever a natural resource is both essential in production¹ and exhaustible, a faster rate of extraction by current generations may imply lower consumption possibilities for future generations. This intergenerational externality is at the heart of the notion of sustainability. If current consumption and the resulting resource extraction pattern do in fact reduce the consumption possibilities of future generations the economy is on a path considered unsustainable. Sustaining consumption across generations requires reduction, through time, of the quantity of resource used per unit of output (resource intensity). Success in achieving that goal will critically depend upon productivity growth and technological possibilities for substituting away from the exhaustible resource in production.

Exhaustible energy (fossil energy plus nuclear energy) is perhaps the most relevant essential exhaustible resource in modern economies. An impressive 92%² of total energy consumption in the world comes from exhaustible sources. This percentage rises to 93.3%³ of total energy consumption in the US (85% from fossil fuels and 8.3% from nuclear). Although renewable sources of energy have gained terrain in the last ten years, with current technologies and despite public efforts, no alternative energy source has shown the potential to replace non-renewable sources at an even moderate scale in the near future.

¹ An input is essential if production collapses to zero whenever the quantity of the input is zero.

² EIA, International Energy Outlook, 2009.

³ EIA, Annual Energy Outlook, 2009.

The American Clean Energy and Security bill of 2009 (ACESA) includes provisions in Titles I and II for the development and adoption of technologies that would reduce (exhaustible) energy intensity. In the electricity sector some of these are biomass-coal co-firing (technologies that produce energy from burning biomass and coal), efficiency gains (technologies that obtain more electricity per unit of exhaustible energy burned), smart grid,⁴ power transmission system upgrades, and combined heat and power systems (systems that both produce efficiency gains and could be fired by biomass). In transportation new technologies include E85, flexible fuel vehicles, and electric vehicles. While all these technologies are supported due to their “energy saving” nature they are bound to affect substitutability of capital and other inputs for exhaustible energy asymmetrically and hence they may have different effects on sustainable consumption.

Whenever an input is both essential in production and exhaustible, the quantity used of that input has to converge asymptotically to zero if a given level of production is to be sustained indefinitely.⁵ We depict technology in Figure 1 by an isoquant describing substitution possibilities between man-made capital and exhaustible energy (i.e. a composite index of exhaustible energy sources) that attain a given level of production. Asymptotic convergence to zero of the quantity of the natural resource used in production is represented by a movement along the isoquant down and to the right.

The ability of society to substitute capital for exhaustible energy in the long run is represented by the asymptotic behavior of the isoquant. The flatter the isoquant in the

⁴ This “modern” version of the grid regulates provision of electricity to buildings, manages provision at peak hours, sets different prices at peak hours, and uses superconductivity to reduce energy loss. It is also thought to incorporate wind and geothermal energy to the distribution system.

⁵ If the quantity does not converge asymptotically to zero then the resource stock will be exhausted in finite time. If it is *equal* to zero (rather than converging asymptotically) then production will be zero due to essentiality.

lower portion the easier it is the substitution. The slope of the isoquant in that portion can be affected by technological progress. The way in which technological progress affects the isoquant depends upon the rate and bias of technical change. In turn these will depend upon the type of technologies whose development and diffusion is being incentivized by energy policies.

Consider, on one hand, deployment of technologies that increase the productivity of energy relative to capital. This type of technical change is captured by a rotation of the isoquant to the left. This in turn would change the isoquant from I^1 to I^2 . Asymptotically is now more difficult to substitute capital for energy making it harder to sustain production in time. On the other hand if technologies that increase the productivity of capital relative to energy are deployed a rotation of the isoquant to the right will occur. This in turn would change the isoquant from I^1 to I^3 . Asymptotically is now easier to substitute capital for energy enhancing the ability of society to sustain production in time.

Additionally suppose a technology that increases the flexibility with which capital and energy can substitute for each other is deployed. This is captured by a flattening of the isoquant (I^4). This change also increases the ability of society to substitute capital for energy asymptotically. Finally consider a technology that increases efficiency of both capital and energy equally without affecting relative productivities. This would be captured by a change from I^1 to I^5 which also increases ease of substitution asymptotically.

It is inferred from Figure 1 that the nature of the technology being incentivized is of importance to determine their effect in asymptotic substitutability between capital and energy and hence in the capacity of society to sustain production in time. Yet no

assessment of the role of different technologies incentivized in ACESA has been conducted so far. The main reason for this is that economic evaluations of ACESA focus on the effectiveness of technologies in reducing emissions from energy rather than their effectiveness in reducing use of energy obtained from exhaustible resources. Therefore an appropriate assessment of the impact of energy policies in sustainability should account for the different effects that incentivized technologies have on substitution possibilities and, hence, sustainable consumption.

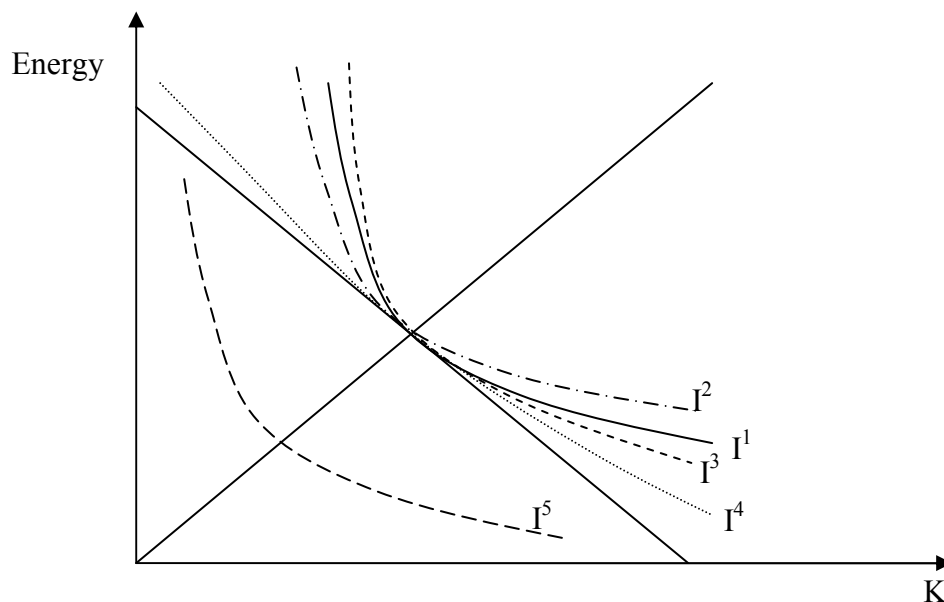


Figure 1. Technical Progress and Changes in Relative Factor Productivities

2. Previous Literature

Previous studies (Dasgupta and Heal 1974; Solow 1974; Stiglitz 1974; Martinet and Doyen 2007) have identified conditions under which an economy could sustain consumption (and ideally *per capita* consumption) indefinitely. Important insights and a rather optimistic view came out of that literature. Three main economic forces capable of offsetting natural resource scarcity were identified: substitution of capital for resource

(through capital accumulation or savings), technological progress, and increasing returns to scale.

These studies based their analysis on specific functional forms representing feasible input-output combinations. More specifically they approximate technological possibilities with a production function displaying constant elasticity of substitution (CES) between capital and natural resource and Hicks neutral technical change. The main results regarding the role of capital accumulation and technological progress can be summarized as follows. Capital accumulation can by itself sustain consumption whenever the elasticity of substitution between capital and energy is greater than one. Neither capital accumulation nor technological progress can sustain consumption if the elasticity of substitution is lower than one. Under unitary elasticity of substitution (Cobb Douglas) capital accumulation can sustain consumption if the elasticity of production with respect to capital is greater than that with respect to natural resource. In addition, if the latter condition holds, technological progress increases the level of sustainable consumption.

There is, however, an important drawback to the assumption of constant elasticity of substitution (CES). Under CES whenever the elasticity of substitution between capital and resource is greater than unity, essentiality of inputs is lost.⁶ As a consequence sustainability is guaranteed.

Therefore the constant substitution assumption may actually be generating a spurious link between substitution and sustainability. One may be induced to believe that whenever the elasticity of substitution is greater than one capital accumulation may be

⁶ The general (1 output – n inputs) CES approximation to technology with elasticity of substitution

different from one is $y = \left[\sum_{i=1}^n \alpha_i x_i^{1/\rho} \right]^\rho$ where ρ represents the elasticity of substitution. It is easily seen from this expression that when an individual input is zero production does not collapse to zero.

enough to sustain consumption and thus that sustainability may be achieved through high savings rates. But in this case sustainability is actually driven by the fact that the resource is not essential in production rather than by a high rate of capital accumulation. Moreover whenever the elasticity of substitution is below unity the economy is doomed and no economic force can offset that. This result seems rather extreme and the natural question one may ask is whether it is robust to the assumption of constant returns to scale.

The literature assuming constant and unitary elasticity of substitution (Solow, Stiglitz, and Martinet et al.), i.e. Cobb Douglas approximation, preserves essentiality and finds both capital accumulation and technological progress to be important in sustaining consumption but faces two problems. A unitary elasticity of substitution is inconsistent with empirical evidence (Koetse 2008) and it rules out (by fixing the elasticity of substitution to a specific value) any interaction between technological progress and substitution. This implies that when using a Cobb Douglas approximation to the technology, policies that induce deployment of technologies such as flexible fuel vehicles and co-fueled power plants that may affect capital-resource substitutability would be ruled out by assumption.

Finally technological progress has been assumed Hicks-neutral⁷ in previous studies, which is problematic for two reasons. First Hicks neutral technical change is inconsistent with available empirical evidence (Sue Wing and Eckaus 2004; Klump 2007). Second if technological progress changes the productivity of capital relative to that of the resource (through technologies such as those discussed in Figure 1) it would affect substitution

⁷ Technological progress is of the Hicks neutral type when it affects total factor productivity without changing relative factor productivities.

rates between energy and other inputs and hence sustainable consumption. This effect would not be captured when technological progress is assumed to be Hicks-neutral.

To sum up the investigation into the role of economic forces in sustainability has been conducted under restrictive technological assumptions. Especially concerning, given the nature of energy policies, is the fact that no insights have come out of these studies regarding the role of biased technical change on sustainability through its effects on both relative productivities and the elasticity of substitution. The present study incorporates a link between technological progress and substitutability by allowing for biased technical change and variable elasticity of substitution. This allows testing of the robustness of previous results in the literature and discussion of potential effects of energy policy (i.e. ACESA) on sustainable consumption under a more flexible framework.

3. The Model

We consider an economy with an essential exhaustible resource. Every period a given quantity of the resource is extracted and used in production. The economy is endowed with a technology that uses capital, a flow of services from the exhaustible natural resource, and other variable inputs to produce an aggregate consumption good. The economy is described by the Solow-Heal-Dasgupta dynamics:

$$\dot{S}(t) = -r(t) \quad (1)$$

$$\dot{K}(t) = f(A, A_r r(t), A_K K(t), A_x x(t)) - c(t) - \lambda K(t) \quad (2)$$

Where $K(t)$ is the stock of human-made capital at time t , $S(t)$ is the level of non-renewable resource stock, $r(t)$ is the flow of the natural resource used in production, $x(t)$

is a vector of other flow inputs used in production in period t (e.g. labor), A is an efficiency factor capturing Hicks-neutral technological progress, A_i is an efficiency factor corresponding to the i^{th} input ($i = r, K, x$) which may increase in time due to technical progress, $f(\cdot)$ is the production function, $c(t)$ is consumption at t and λ is the depreciation rate. Dots above variables denote time derivatives.

The evolution of the exhaustible natural resource is a negative one to one function of the rate of extraction. Zero extraction costs are assumed.

This economy evolves subject to the following constraints:

$$0 \leq r(t) \quad (3) \quad 0 = S_b(t) \quad (4)$$

$$S_b < S(t) \quad (5) \quad c(t) < f(A, A_r r(t), A_K K(t), A_x x(t)) \quad (6)$$

$$0 \leq K(t) \quad (7) \quad 0 < c_b \leq c(t) \quad (8)$$

Where S_b and c_b are arbitrarily chosen values (subscript b denotes “boundary” levels).

The combination of equations (4) and (5) establishes that existing stocks of the resource have to be positive at all times which is a natural condition to impose under essentiality.

From now on the index t will be dropped from the variables for notational simplicity.

The evolution of the stock of human made capital is the difference between output, consumption, and depreciation. The combination of (6) and (8) yields the inequality $c_b < f(A_r r, A_K K, A_x x)$. Plugging this into equation (2) leaves:

$$\dot{K} \leq f(A, A_r r, A_K K, A_x x) - c_b - \lambda K \quad (9)$$

Equation (9) is a differential inclusion denoting the set of all feasible paths for capital evolution.⁸ By solving (9) we find trajectories of resource and capital that are consistent with constraints (3)–(8) at every period t and forever.

A solution to (9) can be obtained exploiting viability theory which was developed by Aubin (1991) and extended to models of growth with exhaustible resources by Martinet and Doyen (2007). Viability theory⁹ does not attempt to choose any "optimal solution" according to given criteria, but selects "viable evolutions" defined as such by ecological, economic, and/or social constraints that state variables are supposed to fulfill. These constraints can be derived from objectives, conservation principles, scientific results of modeling, or precautionary principles. In this economy the main constraint is imposed by technology. The other important constraint is the level of consumption to keep constant. Consumption is fixed at the maximum level that can be sustained through time.

The levels of natural resource S_0 and capital K_0 for which there exists at least one resource extraction profile that sustains consumption at c_b while preventing natural resource exhaustion, form a set called viability kernel. A more technical definition of the viability kernel is (Martinet and Doyen, 2007):

$$Viab(K, x, r, c_b, S_b) = \left\{ (S_0, K_0) \mid \begin{array}{l} \text{there exist decisions } (c(\cdot), r(\cdot)) \text{ and states } (S(\cdot), K(\cdot)) \\ \text{starting from } (S_0, K_0) \text{ satisfying conditions (3)–(8) for any time } t \in \mathbb{R}^+ \end{array} \right\}$$

Emptiness of the viability kernel means that there is no extraction profile that can maintain consumption at c_b without exhausting the natural resource in finite time.

⁸ If the expression maps to a point instead of a set the differential inclusion becomes a differential equation.

⁹ There is by now an established theoretical and empirical literature applying viability theory to economic analysis. Its theoretical foundations can be found in Aubin (1991). Applications of viability can be found in Bene, Doyen, and Gabay (2001), and Doyen and Bene (2003).

Therefore sustainability of c_b is equivalent to non-emptiness of the viability kernel given the technology, and the initial stocks of resources, S_0 and capital K_0 . We discuss now the process by which the viability kernel is derived. The discussion follows Martinet and Doyen closely.

Viability Kernel

Suppose we have a general approximation to the economy's technology denoted by the function $f(A, A_K K, A_r r, A_x x)$. Provided all inputs are essential in production, the resource extraction rate is always positive and, hence, the stock of the exhaustible resource is always decreasing. Moreover, the extraction profile not only reduces the natural resource stock but also affects the capital stock through the equation of motion for capital. Therefore we can express the minimum stock required to sustain a given level of consumption c_b as $V(K, x, S, c_b)$ and its evolution by:

$$\dot{V} + \nabla V(K, x, S, c_b) = \frac{\partial V}{\partial K} \dot{K} + \frac{\partial V}{\partial x} \dot{x} + \frac{\partial V}{\partial S} \dot{S}$$

We will apply this analysis to a data set where the variable input (x) is labor. We will assume in this analysis no population growth and, therefore, we set here $\dot{x}=0$. This assumption does not affect the “sustainable” rate of extraction (see first order condition (10'') below) and it will only affect the expression for sustainable consumption (equation (11)) quantitatively. This is not such a harmful assumption in this context as our main purpose is to explore the qualitative link between technical change and sustainable consumption rather than accurate computation of sustainable consumption.

Assuming that the minimum stock is an autonomous expression (not a direct function of time and, hence, $\dot{V} = 0$) and using the system of equations of motion (1)-(2) and the approximation to the technology we can re-express $\dot{V} + \nabla V(K, x, S, c_b)$ as:

$$\nabla V(K, x, r, c_b) = \frac{\partial V}{\partial K} (f(A, A_r r, A_K K, A_x x) - \lambda K - c_b) - r \quad (10)$$

The boundary of the viability kernel, is characterized by an extraction profile that makes \dot{K} , and \dot{S} , tangent or inward to the viability kernel; i.e. an extraction profile such that $V(\cdot)$ is the solution to the following Hamilton-Jacobi-Bellman (HJB) equation:¹⁰

$$\left\{ \underset{r \in C(K, S, x, c_b)}{\text{Min}} \nabla V(K, x, r, c_b) \right\} = 0 \quad (10')$$

Where $C(K, S, x, c_b) = \left\{ r : c_b < f(A, A_K K, A_r r, A_x x) \text{ and } 0 < S \right\}$

Expression (10') establishes two conditions simultaneously. First $\nabla V(K, x, r, c_b) = 0$ at the optimum and, second, the extraction rate has to minimize $\nabla V(K, x, r, c_b)$. The first order condition (FOC) for the minimization of $\nabla V(\cdot)$ with respect to r yields:

$$\frac{\partial V}{\partial K} = - \frac{1}{f_r(A, A_r r_b, A_K K, A_x x)} \quad (10'')$$

Where $f_r(\cdot)$ is the marginal product of the natural resource and the subscript b in r denotes “boundary” level of r .

¹⁰ The general expression for the HJB is a function V that solves: $\dot{V} + \underset{r \in C(K, S, x, c_b)}{\text{Min}} \nabla V(K, x, r, c_b) = 0$. Since we have assumed V to be autonomous function of time, $\dot{V} = 0$ and equation (10') is obtained. We refer to Aubin (1991, Clark (1995), or Martinet and Doyen (2007) for details.

The expression in (10') is a HJB equation under autonomous function $V(\cdot)$ and it implies that the optimal resource extraction path has to follow FOC (10'') **and** make $\nabla V(K, x, r, c_b)$ equal to zero:

$$c_b = \left(f(\cdot) - \lambda K - r_b / \frac{\partial V}{\partial K} \right) \quad (11)$$

Where subscripts b denote “boundary” levels.

Combining this expression with equation (10'') yields $f(\cdot) - \lambda K + f_r(\cdot)r_b = c_b$, which implicitly defines r_b , the minimum boundary level of resource extraction consistent with a minimum boundary level of consumption. If an explicit expression can be obtained for r_b this could be represented by

$$r_b = G(A, A_r, A_x x, A_K K, c_b; \lambda, \Theta) \quad (11')$$

Where Θ is a vector of technological parameters.

The lower bound of the viability kernel of this economy can be expressed as:¹¹

$$V(K, x, r, c_b, S_b) = \int_{c_b}^{\infty} \frac{1}{f_r(A, G(A_r, A_x x, A_K K, c_b; \lambda, \Theta), A_r, A_K K, A_x x; \Theta)} dK + S_b \quad (12)$$

This expression depicts the minimum level of the resource stock for which there is a trajectory of extraction rates that can keep consumption constant at c_b without exhausting the stock of the resource in finite time.

The viability kernel can now be re-defined as the epigraph¹² of the:

$$V(K, x, r, c_b, S_b): Viab(K, x, r, c_b, S_b) = \{(S, K, x) \text{ such that } S \geq V(K, x, r, c_b, S_b)\}.$$

¹¹ The derivation of the lower bound of the viability kernel involves combining (10') and (11'), integrating both sides, taking limits as k approaches ∞ and rearranging. As a result the minimum stock of the resource, S_b , appears in the equation. This will later be illustrated in proofs of Propositions 1 and 2.

¹² The epigraph of a function is the set of points lying on or above its graph.

From this definition we can see that if $V(c_b) = \infty \quad \forall c_b > 0$ then the viability kernel is empty. Of course it is expected that the higher the level of consumption we want to maintain, the higher the minimum stock of natural resource needed (i.e. $\frac{\partial V(c_b)}{\partial c_b} > 0$) and

hence the three statements below are equivalent:

- There is no positive c_b that can be sustained forever
- $V(K, x, r, c_b, S_b) = \infty \quad \forall c_b > 0$
- $Viab(K, x, r, c_b, S_b) = \emptyset \quad \forall c_b > 0$

Deriving conditions such that $V(K, x, r, c_b, S_b)$ is finite is equivalent to deriving conditions for non-emptiness of the viability kernel and existence of a set of indefinitely sustainable consumption levels $c \leq c_b$. The necessary and sufficient conditions for non emptiness of the viability kernel are those guaranteeing that the integral in the first term of equation (12) converges (i.e. it does not equal ∞) which will depend upon the asymptotic behavior of the integrand. Finally we calculate maximum sustainable consumption by solving the integral in (12) and finding c_b .

To explore the role of technological progress on sustainability the technology is approximated with forms that allow more flexibility, compared to the previous literature, in capturing technical change and substitution possibilities. Then using (12) we find conditions for the existence and level of maximum sustainable consumption and study the impact of biased technical progress and broader substitution possibilities.

4. Technology and Sustainability Analysis with Exhaustible Energy

The rest of the paper uses the theory above with the technological modifications introduced in the analysis of a non-renewable energy resource. The conditions under which a positive sustainable consumption exists are derived, then the maximum sustainable consumption given depletion of stocks of non-renewable energy is obtained. Although we address the case of exhaustible energy we would like to stress the fact that the extensions and conclusions derived in this paper are not confined to the case of energy but are rather applicable to a wider range of exhaustible resources such as the ones mentioned in the introduction.

Previous studies looking at the effect of technological progress and capital accumulation on sustainability approximated technological possibilities with a Cobb Douglas specification $f(A, x, K, E) = Ax^{\alpha_x} K^{\alpha_K} E^{\alpha_E}$ where output y is a function of capital K , a vector of “other inputs” x which will be made explicit soon, non-renewable energy E ,¹³ and an efficiency factor A (usually modeled as exogenous technical change, $e^{\phi t}$) which determines total factor productivity without affecting relative factor productivities (Hicks-neutral technical change). Important parameters describing the properties of this production function are the production elasticity with respect to inputs, the elasticity of scale, and the elasticity of substitution:

- Input production elasticity is defined as the percentage change in output after a one percent change in the level of the input. Capital elasticity of production is $\frac{d \ln f(.)}{d \ln k} = \alpha_K$

and energy elasticity of production is $\frac{d \ln f(.)}{d \ln E} = \alpha_E$.

¹³ This is equivalent to the rate of extraction r from our previous generic analysis.

- The elasticity of scale is the percentage change in output after a proportional change

in the level of all inputs, $\left. \frac{d \ln (f(\lambda K, \lambda E, \lambda x))}{d \ln \lambda} \right|_{\lambda=1} = \alpha_K + \alpha_E + \alpha_X$

- The (Morishima)¹⁴ elasticity of substitution (MES) between capital and energy is a measure of the ease with which energy can be substituted for capital in production and is

expressed as: $\sigma_{KE}^M = \frac{f_E}{K} \frac{F_{KE}}{F} - \frac{f_K}{E} \frac{F_{KE}}{F}$ where F is the determinant of the bordered

Hessian of the production function, f_E is the marginal productivity of energy flow, f_K is the marginal productivity of capital, E is the energy flow used in production, K is capital used in production and F_{KE} is the cofactor associated with f_{KE} which is the derivative of the marginal productivity of capital with respect to energy. $\sigma_{KE}^M > 0$ means the inputs are substitutes and $\sigma_{KE}^M < 0$ means they are complements. In this particular case the elasticity of substitution is constant and equal to 1.

- Rate of Hicks-neutral technological change: $\frac{d \ln y}{dA}$.

As can be seen from the parameters above the technological factor A does not enter into the expressions for the capital or energy production elasticities. Therefore technological progress does not affect inputs' relative productivity. Likewise A does not affect the elasticity of substitution. Therefore there is no role for technological progress in substitutability in this context. Under this approximation to technology previous literature has found that without technological progress ($\phi = 0$) a positive sustainable consumption exists whenever the capital elasticity of production is greater than that of energy

¹⁴ We use Morishima elasticities instead of Allen elasticities. It is argued in the literature (Blackorby and Russell (1989)) that the Morishima elasticity is "superior" because: it is asymmetric, it is an exact measure of ease of substitution, and it provides complete comparative static information about relative factor shares.

($\alpha_K > \alpha_E$). Moreover sustainable consumption, if it exists, increases with positive technological progress ($\phi > 0$). While technological progress can increase sustainable consumption, its role in existence was not explored.

To overcome the limitation that technological progress does not affect relative factor productivity and to explore its role in existence we introduce the possibility of biased technological progress, i.e. technological progress that affects the capital-energy relative factor productivity. We will consider the case where technological progress may also affect the elasticity of substitution afterwards.

4.1. Technological Progress and Relative Productivities: Introducing Biased Technical Change

We maintain in this section the assumption of constant and unitary elasticity of substitution but introducing the possibility of biased technological progress. An expression for a Cobb Douglas function with factor-augmenting technical change is:

$$f(A, x, K, E) = A (A_x x)^{\alpha_x} (A_K K)^{\alpha_K} (A_E E)^{\alpha_E} \quad (13)$$

For the purpose of this study we will assume $A_x = 1$,¹⁵ $A_K = K^{\frac{\varepsilon}{\alpha_K}}$ and $A_E = E^{\frac{\gamma}{\alpha_E}}$.

Hence the productivity of capital and energy depend upon their own accumulated levels. The parameters ε and γ capture innovations increasing the efficiency of capital and energy respectively while α_K and α_E are assumed constant. Therefore a higher diffusion of technologies such as first and second generation biofuels, biomass and fossil co-fueled gasification, biomass-fired combined heat and power, wind, and solar would increase ε .

¹⁵ Therefore there is no technological progress increasing the efficiency of inputs other than capital and energy.

On the other hand higher diffusion of fossil-fired combined heat and power, smart grid, and other technologies achieving efficiency gains in the use of exhaustible energy would increase γ .

This specification is consistent with the type of technical change derived in models of endogenous growth by Acemoglu (2002a) and has three main advantages: it is flexible in the sense that it allows for non-neutral technical change, it is able to generate empirical regularities,¹⁶ and it is consistent with empirical evidence for the U.S. offered by Wing and Eckaus using an extended version of Jorgenson's KLEM data.¹⁷

Parameters describing the properties of this production function are as follows:

- Input production elasticity is defined as the percentage change in output after a one percent change in the level of the input. Capital elasticity of production is

$$\frac{d \ln f(.)}{d \ln k} = \alpha_K + \varepsilon \text{ and energy elasticity of production is } \frac{d \ln f(.)}{d \ln E} = \alpha_E + \gamma.$$

- The capital and energy relative factor productivity is depicted by the marginal rate of

substitution between both inputs, $MRS_{KE} = \frac{f_K}{f_E} = \frac{(\alpha_K + \varepsilon) E}{(\alpha_E + \gamma) K}$

- Elasticity of Scale: $\alpha_K + \varepsilon + \alpha_E + \gamma + \alpha_X$
- Elasticity of substitution: constant and equal to 1

¹⁶ Acemoglu (2002b) stresses 4 important empirical regularities: 1) the past sixty years have seen a large increase in the supply of more educated workers, while returns to education have risen; 2) returns to education fell during the 1970s, when there was a very sharp increase in the supply of educated workers. Returns to education then began a steep rise during the 1980s; 3) Overall wage inequality rose sharply beginning in the early 1970s. Increases in within-group (residual) inequality, account for much of this rise; 4) Average wages have stagnated and wages of low-skill workers have fallen in real terms since 1970.

¹⁷ Estimations in that paper revealed strong evidence in favor a non neutral embodied technological progress.

○ Rate of technological change: $RTC = (\ln K)d\varepsilon + (\ln E)d\gamma + d\ln(A)$. If we start at $\varepsilon = \gamma = 0$, then $d\varepsilon = \varepsilon$ and $d\gamma = \gamma$ and we can re-express the rate of technical change in terms of ε and γ : $RTC = (\ln K)\varepsilon + (\ln E)\gamma + d\ln(A)$.

○ Input bias of technological change (IBTC). Technical change is capital (energy) augmenting if $IBTC = \frac{d\varepsilon}{(\alpha_K + \varepsilon)} - \frac{d\gamma}{(\alpha_E + \gamma)} > (<)0$. Then, if we start at $\varepsilon = \gamma = 0$,

$$IBTC > (<)0 \Leftrightarrow \varepsilon > (<) \frac{\alpha_K}{\alpha_E} \gamma.$$

In contrast to the previous approximation to technology, here technological progress affects both the capital elasticity of production and the energy elasticity allowing for biased technical change. We will now derive conditions for existence of a positive sustainable consumption and obtain an expression for maximum sustainable consumption. From these we will be able to infer the role of biased technical change in sustainability.

First we proceed to derive the viability kernel of this economy approximating technological possibilities with (13) and assuming zero capital depreciation.¹⁸

PROPOSITION 1. Consider an economy using exhaustible energy, with no capital depreciation ($\lambda = 0$) and a CD technology with endogenous factor augmenting technical change $f(\cdot) = A x^{\alpha_x} K^{\alpha_K + \varepsilon} E^{\alpha_E + \gamma}$ where $\varepsilon, \gamma \geq 0$. Then the viability kernel depends on parameters as follows:

¹⁸ Results for existence of a positive sustainable consumption under positive capital depreciation have been derived and are available from the authors.

$$Viab(K, x, r, c_b, S_b) = \begin{cases} \{(S, K) \text{ such that } S \geq V(K, c_b, S_b)\} & \text{if } (\alpha_K + \varepsilon) > (\alpha_E + \gamma) \\ & \text{and } (\alpha_E + \gamma) < 1 \\ \emptyset & \text{otherwise} \end{cases}$$

Where $V(K, x, r, c_b, S_b)$ is a function defined by:

$$V(K, x, r, c_b, S_b) = \frac{A x^{\frac{-\alpha_K}{\alpha_E + \gamma}}}{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)} \left(\frac{C_b}{[1 - (\alpha_E + \gamma)]} \right)^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} K^{\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{\alpha_E + \gamma}} + S_b$$

Proof: Appendix A.1.

A capital elasticity of production greater than that of energy, $(\alpha_K + \varepsilon) > (\alpha_E + \gamma)$, and a sufficiently low energy-augmenting technical change, $(\alpha_E + \gamma) < 1$, are necessary and sufficient for non emptiness of the viability kernel or, equivalently, existence of a positive sustainable consumption. Therefore capital augmenting technical change, in contrast to Hicks-neutral technical change, can guarantee existence of a positive sustainable consumption. In particular if $\alpha_E - \alpha_K < \varepsilon - \gamma < 1$ then existence is guaranteed. Finally the elasticity of scale plays no role in existence. The mechanics behind this result may be better understood by looking at our illustration in Figure 1. Capital augmenting technical change tends to rotate the isoquant to the right reducing the slope in the inferior portion of the isoquant (i.e. when energy tends to zero) therefore reducing the ability of society to substitute capital for energy asymptotically.

Let us define the maximum level of consumption (MSC) that can be sustained in this economy as the maximum constant consumption for which current stocks of capital and resource belong to the viability kernel:

$$c^* = \max(c_b : (K_0, S_0, x_0) \in Viab(K, x, r, c_b, S_b))$$

Combining this definition with the viability kernel derived in Proposition 1 yields the following expression for maximum sustainable consumption:

$$c_{CD}^* = \frac{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)}{(\alpha_E + \gamma)(1 - (\alpha_E + \gamma))} (\alpha_E + \gamma)^{\frac{\alpha_E + \gamma}{1 - (\alpha_E + \gamma)}} A^{\frac{1}{1 - (\alpha_E + \gamma)}} x^{\frac{\alpha_x}{1 - (\alpha_E + \gamma)}} S_0^{\frac{\alpha_E + \gamma}{1 - (\alpha_E + \gamma)}} K^{\frac{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \quad (14)$$

From the expression above we can infer several important conclusions regarding the role of different economic forces in sustainability. The expression for c_{CD}^* shows that maximum sustainable consumption, provided existence conditions are met, increases with Hicks-neutral technological progress, which is consistent with previous results by Stiglitz (1974). Maximum sustainable consumption also increases with capital augmenting technical change. These results suggest that a higher diffusion of first and second generation biofuels, biomass-fired energy production, wind, and solar (increases in ε) would increase maximum sustainable consumption. On the other hand higher diffusion of technologies achieving efficiency gains in the use of exhaustible energy such as fossil-fired combined heat and power, and smart grid (increases in γ) may harm sustainability by either increasing likelihood of inexistence of a positive sustainable consumption or reducing maximum sustainable consumption. This is due to the fact that these technologies increase the productivity of energy relative to other inputs reducing the ability of society to substitute capital for energy.

Moreover maximum sustainable consumption increases with resource endowment (S_0), and non energy inputs, x and K . This implies that higher savings rate and capital accumulation enhance sustainability. In particular the elasticity of MSC with respect to capital is $\frac{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}$ implying that the more productive capital is relative to

energy the higher the impact of savings and capital accumulation on sustainable consumption. Given the way in which $(\alpha_K + \varepsilon)$ and $(\alpha_E + \gamma)$ enter c_{CD}^* we can not state that a higher elasticity of scale would enhance sustainability. This is in contrast with the positive effect of elasticity of scale on sustainable consumption found by Stiglitz (1974).

The analysis above has overcome one drawback of previous literature, Hicks-neutral technological progress. There is still however a potentially important limitation, the assumption of constancy of the elasticity of substitution which rules out any effect of technological progress on this parameter. We will now expand the analysis to overcome this limitation.

4.2. Technological Progress and Substitution: Introducing Variable Elasticity of Substitution

We are interested in approximating the technology with a production function that allows for variable elasticity of substitution and essentiality of inputs. The following transcendental specification (using the same type of technological progress as before) fulfills both properties:¹⁹

$$f(.) = A x^{\alpha_x} K^{\alpha_K + \varepsilon} E^{\alpha_E + \gamma} \exp\left(\delta_k K^{\frac{\alpha_K + \varepsilon}{\alpha_K}}\right) \exp\left(\delta_E E^{\frac{\alpha_E + \gamma}{\alpha_E}}\right) \exp(\delta_x x) \quad (15)$$

The exponential portion of the expression allows for variable substitution while at the same time keeping essentiality of all inputs. As it turns out, the transcendental approximation yields analytical solutions only if production is log-linear in energy

¹⁹ The transcendental approximation has been chosen from a complete list of specifications reviewed by Griffin et al. There they summarized functional forms used in empirical work and discussed their properties.

(i.e. $\delta_E = 0$). We will discuss the implications of this constraint as we derive the parameters that describe features of this technology.

Parameters we use to describe important features of technology in the transcendental specification are:

- Capital elasticity of production is $\frac{d \ln f(.)}{d \ln K} = \left[\alpha_k + \varepsilon + \frac{\delta_k (\alpha_k + \varepsilon)}{\alpha_K} K^{\frac{\alpha_K + \varepsilon}{\alpha_K}} \right]$ and energy

elasticity of production is $\frac{d \ln f(.)}{d \ln E} = \alpha_E + \gamma$. Note that the capital production elasticity is a function of K but the energy production elasticity is not a function of E due to the assumption that output is a log-linear function of energy ($\delta_E = 0$). Therefore this assumption implies constant energy production elasticity.

- Elasticity of Scale is $\alpha_X + (\alpha_K + \varepsilon) + (\alpha_E + \gamma) + \delta_X x + \left(\delta_k \frac{(\alpha_K + \varepsilon)}{\alpha_K} K^{\frac{\alpha_K + \varepsilon}{\alpha_K}} \right)$. The

elasticity of scale is not affected by the level of energy used due to the assumption $\delta_E = 0$.

- Elasticity of substitution. The analytical expression for the capital-energy Morishima elasticity of substitution (σ_{KE}^M) is cumbersome in this case. However under this specification, it is an algebraic matter to show that $\sigma_{KE}^M > (<) 1$ if and only if $\delta_k > (<) 0$,

and $\frac{\partial \sigma_{KE}^M}{\partial \alpha_K} < 0$ and $\frac{\partial \sigma_{KE}^M}{\partial \alpha_E} > 0$.

- The rate of technological change is: $RTC = d \ln(A) + (\ln k) \varepsilon + (\ln E) \gamma + \frac{\delta_K}{\alpha_K} \ln(K) K \varepsilon$.

A relaxation of the assumption $\delta_E = 0$ would change the shape of the relationship between RTC and energy (E).

- Technical change is capital (energy) biased if

$$IBTC = \left(\frac{K^{-1} + (\delta_k / \alpha_K)(1 + \ln(K))}{\alpha_K K^{-1} + \delta_k} \right) \varepsilon - \frac{\gamma}{\alpha_E} > (<) 0. \text{ This is constant with respect to } E \text{ due}$$

to the assumption $\delta_E = 0$.

Based on parameters above we infer that this approximation to the technology not only allows for non-neutral technological progress but it permits technological progress to affect the elasticity of substitution. Just as with the Cobb Douglas approximation we will now derive conditions for the existence of a positive sustainable consumption (Proposition 2), obtain an expression for maximum sustainable consumption, and discuss the role of different economic forces on sustainability.

PROPOSITION 2. Consider an economy using exhaustible energy, with no capital depreciation ($\lambda = 0$) and a transcendental technology with endogenous factor augmenting

technical change $f(.) = A x^{\alpha_X} K^{\alpha_K + \varepsilon} E^{\alpha_E + \gamma} \exp\left(\delta_K K^{\frac{\alpha_K + \varepsilon}{\alpha_K}}\right)$ where $\varepsilon, \gamma \geq 0$. Then the

viability kernel depends on parameters as follows:

$$Viab(K, x, r, c_b, S_b) = \begin{cases} \{(S, K) \text{ such that } S \geq V(K, c_b, S_b)\} & \text{if } \alpha_K + \varepsilon < \alpha_E + \gamma \text{ and } \delta_K > 0 \\ \emptyset & \text{otherwise} \end{cases}$$

Where $V(K, x, r, c_b, S_b)$ is a function defined by:

$$V(K, x, r, c_b, S_b) = \left[(\alpha_E + \gamma) A \right]^{-\frac{1}{(\alpha_E + \gamma)}} x^{-\frac{\alpha_X}{\alpha_E + \gamma}} \left[\frac{(\alpha_E + \gamma)}{\delta_K} \right]^{\frac{\alpha_K}{\alpha_K + \varepsilon} \left[1 - \frac{\alpha_K + \varepsilon}{(\alpha_E + \gamma)} \right]} C b^{\frac{1 - (\alpha_E + \gamma)}{\alpha_E + \gamma}} \frac{\alpha_K}{\alpha_K + \varepsilon} \Gamma(a + 1, F(K)) + S_b$$

Where $\Gamma[a+1, F(K)]$ is the upper incomplete gamma function²⁰ with

$$F(K) = \left[\frac{\delta_K}{(\alpha_E + \gamma)} K^{\frac{(\alpha_K + \varepsilon)}{\alpha_K}} \right]^{-\frac{\alpha_K}{(\alpha_E + \gamma)} - \frac{\varepsilon}{(\alpha_K + \varepsilon)}} \text{ as the lower limit of integration.}$$

Proof: see Appendix A.2.

An energy elasticity of production greater than that of capital, $(\alpha_E + \gamma) > (\alpha_K + \varepsilon)$, and $\delta_K > 0$ are necessary and sufficient conditions for non emptiness of the viability kernel or, equivalently, existence of a positive sustainable consumption. This is a somewhat surprising result for two reasons. First technological progress, regardless of rate and bias, is not sufficient to guarantee existence. Second, in contrast to the Cobb Douglas case in which capital-augmenting technical change enhanced sustainability, it is energy-augmenting technical change that increases the likelihood of existence. Therefore allowing technological progress to affect capital-energy elasticity of substitution results in conclusions opposite to those obtained under a constant elasticity of substitution. The causes of this reversal will be explored later in our numerical simulation.

Combination of our definition of MSC with the viability kernel derived in Proposition 2 yields the following expression for maximum sustainable consumption:

$$c_{TRA}^* = A^a \left[\frac{(\alpha_K + \varepsilon)}{\alpha_K} \right]^b (\delta_K)^c [1 - (\alpha_E + \gamma)]^a (\alpha_E + \gamma)^b x^b S_0^b (\Gamma[d, K])^{-b} \quad (16)$$

$$\text{Where: } a = \frac{1}{1 - (\alpha_E + \gamma)}, \quad b = \frac{(\alpha_E + \gamma)}{1 - (\alpha_E + \gamma)}, \quad c = \frac{\alpha_K}{(\alpha_K + \varepsilon)} \left[\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{1 - (\alpha_K + \varepsilon)} \right], \quad d = \frac{\alpha_K [(\alpha_E + \gamma) - (\alpha_K + \varepsilon)]}{(\alpha_K + \varepsilon)(\alpha_E + \gamma)}$$

²⁰ For a complex number z with positive real part the Gamma function is defined by $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$.

The upper incomplete gamma function is a particular case of the gamma function in which the lower limit of integration is different from zero: $\Gamma(z, x) = \int_x^\infty t^{z-1} e^{-t} dt$.

Given the complicated nature of technological parameters it is hard to draw conclusions on the role of different forces in sustainability. However it is clear from (16) that, in addition to conditions derived in Proposition 2, a positive sustainable consumption exists if $(\alpha_E + \gamma) < 1$. This is consistent with the Cobb Douglas case in the sense that a high energy augmenting technical change, i.e. $\gamma \geq 1 - \alpha_E$, may harm sustainability. Moreover maximum sustainable consumption increases with Hicks-neutral technological progress, with resource endowment (S_0), and with non energy inputs, x . The role of biased technological progress on maximum sustainable consumption is not clear from this analytical expression and will be addressed later through numerical simulations.

Finally existence requires $\delta_K > 0$ which, as mentioned before, implies a capital-energy elasticity of substitution greater than one. Under CES an elasticity of substitution greater than one was sufficient for existence while under variable elasticity of substitution (VES) it is necessary. This is not surprising once one realizes that the approximation with VES preserves essentiality of the resource while this is not the case with a CES approximation. This result confirms our suspicions that whenever essentiality of inputs is preserved more substitutability is required to guarantee existence of a positive sustainable consumption.

5. Computation of Maximum Sustainable Consumption for the U.S. Economy

We estimate the parameters of the technology based on Jorgenson's KLEM (capital, labor, energy²¹ and materials) data set for the U.S. economy (Jorgenson 2007).²² This

²¹ Energy includes coal mining, petroleum & gas mining, petroleum refining, electric utilities, and gas utilities. Although Jorgenson has data on total energy rather than non-renewable energy sources, when we compare this data to the data from the DOE, non-renewable energy accounts for 93.3% of the shares in total energy used in production. Assuming all energy used is non-renewable is not extremely harmful.

²² Available online at:

data consist of a panel covering 35 sectors of the US economy from 1960 to 2005. To estimate the aggregate technology in this economy we construct aggregate indexes of inputs (capital, labor, and energy) and output (material) in each year. Specifically, we aggregate inputs and output across 35 sectors in each year using a Cobb Douglas aggregator function where coefficients are given by the share of each individual element on total value.

The technology of this economy may be described by a production possibilities frontier combining all four aggregates. We further assume separability between materials and the rest of the elements which allows us to model the aggregate technology with a production function where production of materials²³ is a function of primary inputs (labor and capital) and intermediate inputs (energy).²⁴ Separability across inputs and output implied by aggregation procedures and existence of an aggregate production function may bias estimations. However it is not our purpose here to find the best fit for the US aggregate technology but rather explore the role of different types of technological progress on MSC and assuming existence of an aggregate production function greatly simplifies our illustration.

We fit a Cobb Douglas and transcendental approximations to these data. The estimated parameters serve as basis for testing sustainability conditions derived in Propositions 1 and 2 and also provide a plausible starting point around which to simulate changes in technology. Parameter estimates were obtained with MATLAB²⁵ and the results are

http://dvn.iq.harvard.edu/dvn/dv/jorgenson/faces/study/StudyPage.jsp?studyId=18782&tab=files&studyListingIndex=2_34b20438e7071dea36e7852bed3f

²³ This aggregate includes production from agriculture, metallic and non-metallic materials, services, textile-apparels, wood-paper, other services, other metals, machinery, and equipment.

²⁴ Existence of an aggregate production function implies that production functions of all sectors are identical up to a scalar multiple (Denny (1972) and Hall (1973)).

²⁵ Programs are included in Appendix B.

reported in Appendix B. We have tested conditions for existence of a positive sustainable consumption under both approximations. Existence conditions with a CD specification ($\alpha_K + \varepsilon > \alpha_E + \gamma$ and $\alpha_E + \gamma < 1$ from Proposition 1) could not be rejected. The impact of different types of technical change on MSC is clear (it increases (decreases) with capital (energy) augmenting technical change) from equation (14) and so, there is no need for numerical simulation.

On the other hand existence conditions ($\alpha_K + \varepsilon > \alpha_E + \gamma$ and $\alpha_E + \gamma < 1$ from Proposition 2) are rejected at 1% level of significance when technology is approximated with a transcendental specification.²⁶ This in turn implies that maximum sustainable consumption is zero.

The goal of this study is to explore the link between maximum sustainable consumption and different types of technological progress better captured by a transcendental approximation. This link is not clear from equation (16) and so numerical simulations are needed. To be able to simulate maximum sustainable consumption we will impose sustainability ($\alpha_K + \varepsilon < \alpha_E + \gamma < 1$) by setting $\alpha_E = 0.3$, which is higher than 0.2724, the estimated value of α_K . After imposing sustainability we use equation (16) to simulate different rates and directions of technical change and maximum sustainable consumption.

Computation of MSC with a transcendental approximation (c_{TRA}^*) is conducted for the year 2005. The stock of non renewable energy S_0 was calculated with data from the US

²⁶ Since the CD specification is nested into the transcendental we conducted an F-test of the hypothesis $\delta_K = 0$ (i.e. the restriction that would support a CD approximation). The hypothesis was rejected at the 0.01 level of significance which offers support to the use of transcendental approximation over a CD (but not necessarily over other more flexible specifications) to model this economy.

Department of Energy (DOE).²⁷ The procedure to calculate S_0 consists of constructing an index aggregating all non-renewable energy sources (measured in Million British Thermal Units, MBTUs). For consistency with our previous aggregation procedure we calculate a Cobb Douglas function of the different energy sources where coefficients for each source are shares in total MBTUs used in production.²⁸ This of course implies that we are calculating MSC based on US own resources and that total non-renewable energy used annually in production is extracted from US reserves and no imports occur. This is far from truth since almost 70% of non-renewable energy used by the US is imported. However the issue we are addressing here is the maximum level of consumption that can be sustained by the US economy with its own energy resources.

6. Technological Progress, Substitutability and MSC: Transcendental Approximation

Our goal in this section is to isolate the effect of Hicks-neutral technical change (RTC), the input bias of technical change (IBTC) and of the elasticity of substitution (ES) on MSC. Three simulations are conducted within the framework of the transcendental specification, aiming at isolating the effects of technological change and substitutability on MSC. Simulations were conducted using the mesh routine in MATLAB.

Figure 2 captures the relationship between the Hicks-neutral rate of technical change²⁹ (RTC) and MSC. An increase in RTC increases MSC. The sensitivity of MSC to RTC is good news for the economy. This result calls for optimism in the analysis of

²⁷ Stock of different sources of energy can be found in <http://www.eia.doe.gov/>.

²⁸ Results are reported in Appendix D.

²⁹ Changes in Hicks-neutral rate of technical change are achieved through changes in A keeping everything else constant.

sustainability but we should keep in mind that while Hicks-neutral technical change can increase MSC it can not guarantee existence. An example of such a change would be an upgrade of the power transmission system regardless of the energy source.

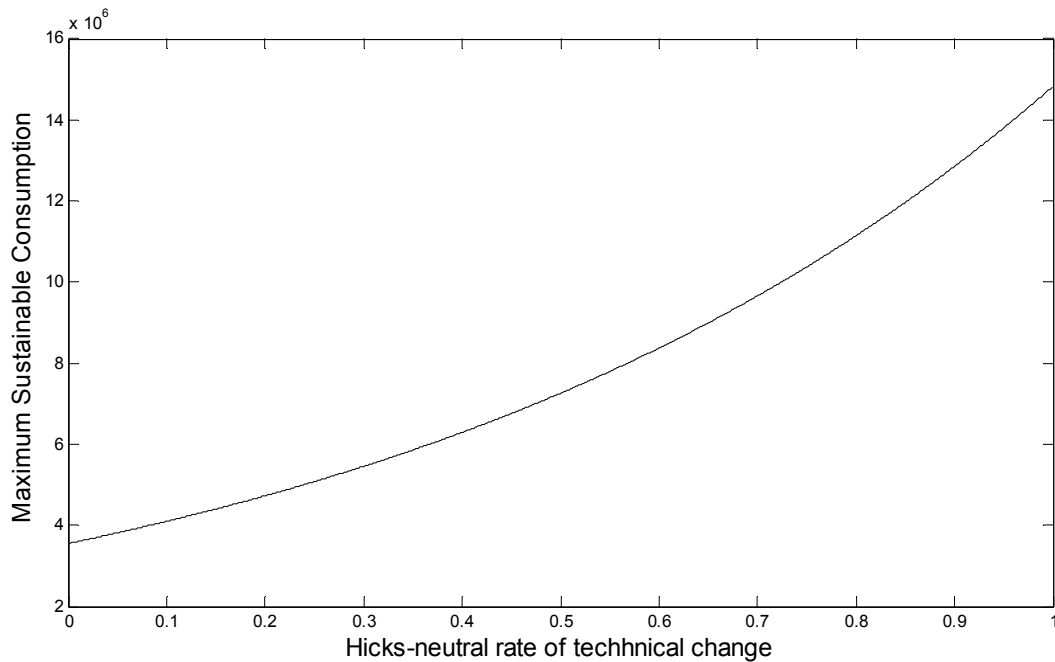


Figure 2. Neutral Technical Change and MSC (Transcendental)

Figure 3 depicts the relationship between MSC and non-neutral technical change with a transcendental approximation. Consistent with the CD case, capital-augmenting technical change increases MSC. However, in contrast to the CD case, an energy-augmenting technical change also increases MSC.

The intuition behind this result is easily understood looking at Figure 4. Although energy-augmenting technical change, by increasing the productivity of energy relative to that of capital, may tend to reduce the ability of society to substitute capital for energy, it also enhances substitutability asymptotically by flattening the isoquant between capital

and energy. The latter effect is strong enough to outweigh the former rendering the positive relationship between energy-augmenting technical progress and MSC.

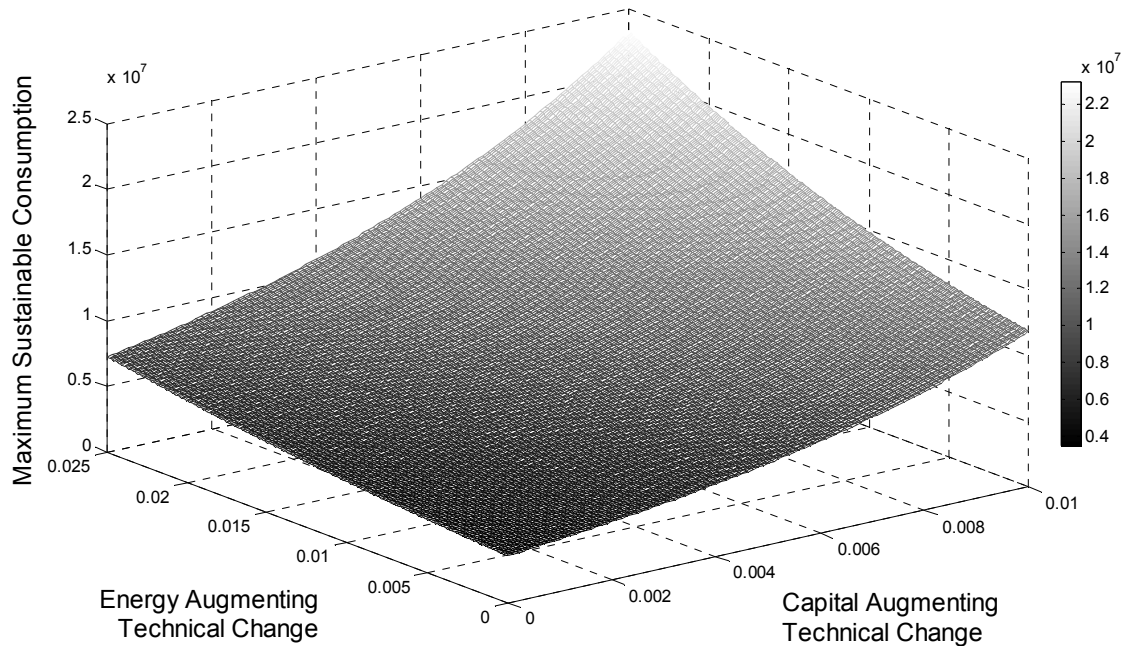


Figure 3. Non Neutral Technical Change and MSC

The effect of technical change on substitutability is neglected in the CD specification (due to constancy of elasticity of substitution) yielding a negative relationship between energy-augmenting technical progress and MSC. Therefore this result clearly illustrates how the inflexibility of a CD specification handicaps our understanding of sustainability since it neglects the wide range of substitutions usually displayed by real economies. Therefore development and higher diffusion of technologies such as fossil-fired combined heat and power, smart grid, and other technologies achieving efficiency gains in the use of exhaustible energy may in fact enhance sustainability.

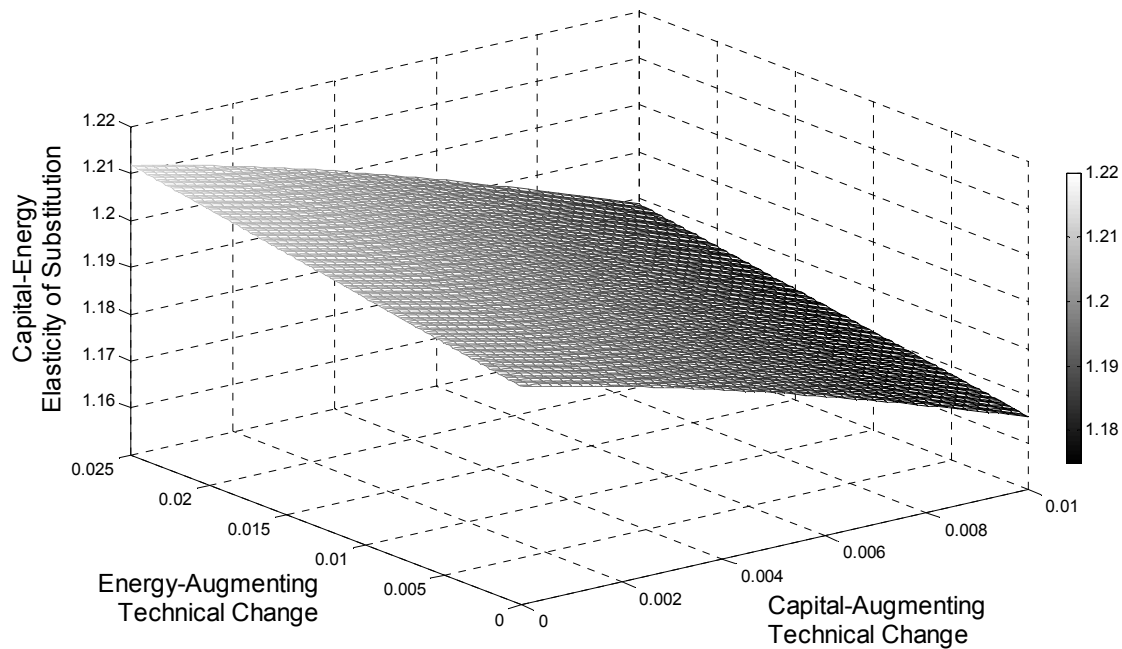


Figure 4. Non Neutral Technical Change and Capital-Energy Substitution

7. Summary and Conclusions

Policies aimed at reducing use of fossil energy per unit of output incentivize the deployment of new technologies that may affect the productivity of exhaustible energy relative to that of other inputs, most importantly capital, that substitute for it. This in turn may affect the ability of society to reduce the use of energy and sustain consumption in the long run. A clear understanding of the effect of new technologies on substitution possibilities and the resulting impact on sustainable consumption is critical in evaluating these policies. Previous studies of sustainable consumption ruled out any effect of technological progress on relative productivities and substitution elasticities which handicaps evaluation of energy policies. This paper is an attempt to correct this problem by allowing for biased technological progress and variable elasticity of substitution.

Allowing technological progress to affect relative factor productivities (biased technical change) within a Cobb Douglas approximation to technology has rendered new insights. Only capital augmenting technological change can guarantee the existence of a positive sustainable consumption. Moreover capital augmenting technical change increases maximum sustainable consumption while resource augmenting technical change decreases it. Finally although Hicks-neutral technical change can not achieve existence, it increases maximum sustainable consumption provided a positive sustainable consumption exists. This suggests that more resources should be allocated to development and diffusion of technologies such as first and second generation biofuels and other sources of renewable energy.

We have taken the analysis one step further by allowing for technological progress to affect substitution possibilities between capital and energy. This was achieved by approximating technology with a transcendental production function. This extension yielded results that reverse those previously derived in the literature. In particular a capital-energy elasticity of substitution greater than one is a necessary condition for existence of a positive sustainable consumption in contrast to the CES approximation for which it is sufficient. This indicates that when inputs are essential in production more substitutability is required to achieve sustainability. Moreover with flexible substitution, resource augmenting technical change increases the likelihood of existence of a positive sustainable consumption. This reverses the conclusion obtained when using a Cobb Douglas approximation which indicated that capital augmenting technical change enhanced sustainability. This is due to the positive effect of energy augmenting technical change on capital-energy elasticity of substitution (i.e. the curvature of the isoquant),

effect neglected under constant elasticity of substitution. This result suggests that, in addition to technologies mentioned above, resources should also be allocated to technologies such as fossil-fired combined heat and power and smart grid.

Finally since technological progress may enhance or damage sustainability depending on its effect on substitution possibilities more research aimed at a better understanding and measurement of the impact of technological progress on substitution possibilities is needed.

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Appendix A.A. Proof of Proposition I.

With a Cobb Douglas approximation the production function can be denoted by:

$$f(A, K, L, E) = A (K)^{\alpha_K + \varepsilon} (L)^{\alpha_L} (E)^{\alpha_E + \gamma}$$

Therefore:

$$\frac{\partial V}{\partial K} = -\frac{1}{f_E} = \frac{1}{(\alpha_E + \gamma) E_b^{-1} (A) (K)^{\alpha_K + \varepsilon} (E_b)^{\alpha_E + \gamma} (L)^{\alpha_L}} \quad (1)$$

Where $f_E(\cdot)$ is the marginal product of energy and the subscript b in E denotes “boundary” level of E .

At the minimum (c_b, E_b) , $\frac{\partial V}{\partial K} (f(\cdot) - c_b - \lambda K) + E_b = 0$. Hence:

$$E_b = \left(f(\cdot) - \lambda K - r_b / \frac{\partial V}{\partial K} \right) \quad (2)$$

Plugging (1) in (2) yields:

$$E_b = \left[\frac{c_b + \lambda K}{1 - (\alpha_E + \gamma)} \right]^{\frac{1}{(\alpha_E + \gamma)}} K^{-\frac{(\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \quad (3)$$

Our viability kernel is depicted by:

$$V(K, c_b, S_b) = \int_K^{\infty} \frac{1}{f_E(A, A_E E_b(c_b), A_K K, A_L L)} dx + S_b \quad (4)$$

Plugging (3) in (1) and the resulting expression in (4) yields:

$$V(K, c_b, S_b) = A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \frac{1}{(\alpha_E + \gamma)} \left[\frac{c_b}{1 - (\alpha_E + \gamma)} \right]^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \int_K^{\infty} x^{-\frac{(\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} dx + S_b$$

Solving the integral yields:

$$V(K, c_b, S_b) = \frac{A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}}}{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)} \left[\frac{c_b}{1 - (\alpha_E + \gamma)} \right]^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \left[\infty^{\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} - K^{\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} \right] + S_b$$

Therefore the minimum level of resource stock needed to sustain consumption at c_b (i.e.

$V(K, c_b, S_b)$) will be finite if only if $\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}$ is negative. This will in turn hold

whenever $(\alpha_K + \varepsilon) > (\alpha_E + \gamma)$. Provided this condition holds the integral can be solved and the viability kernel can be expressed as:

$$V(K, c_b, S_b) = \frac{A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}}}{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)} \left(\frac{c_b}{[1 - (\alpha_E + \gamma)]} \right)^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} K^{\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{\alpha_E + \gamma}} + S_b \quad (5)$$

But then again $V(K)$ will be positive and finite if and only if $(\alpha_E + \gamma) < 1$.

Summarizing, any positive level of consumption c_b is sustainable if the resource stock required to sustain it indefinitely is finite. This will in turn be true if and only if the viability kernel is not empty for c_b . Conditions for this are $(\alpha_E + \gamma) < 1$ and

$\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_E + \gamma)} < 0$ and, therefore, we call them non-emptiness conditions or NEC ■

Maximum Sustainable Consumption

The maximum level of sustainable consumption will be achieved if we allow the stock of the natural resource to converge to zero asymptotically. In addition to compute this level we need to find the c_b that corresponds to the existing stock of natural resource S_0 .

Therefore we set $V(K, c_b, S_b = 0) = S_0$ and the viability kernel can now be denoted as:

$$S_0 = \frac{A L^{\frac{\alpha_L}{(\alpha_E + \gamma)}}}{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)} \left(\frac{c_b}{[1 - (\alpha_E + \gamma)]} \right)^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} K^{\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{\alpha_E + \gamma}} + 0 \quad (6)$$

Solving for c_b in equation (6) yields:

$$c_{CD}^* = S_0^{\frac{(\alpha_E + \gamma)}{1 - (\alpha_E + \gamma)}} A^{\frac{1}{1 - (\alpha_E + \gamma)}} L^{\frac{\alpha_L}{1 - (\alpha_E + \gamma)}} \left[(\alpha_K + \varepsilon) - (\alpha_E + \gamma) \right]^{\frac{(\alpha_E + \gamma)}{1 - (\alpha_E + \gamma)}} \left[1 - (\alpha_E + \gamma) \right] K^{\frac{(\alpha_K + \varepsilon) - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}}$$

Appendix A.B. Proof of Proposition II.

With a transcendental approximation the production function can be denoted by:

$$f(.) = A L^{\alpha_L} K^{\alpha_K + \varepsilon} E^{\alpha_E + \gamma} \exp \left(\delta_k K^{\frac{\alpha_K + \varepsilon}{\alpha_K}} \right)$$

Therefore:

$$\frac{\partial V}{\partial K} = - \frac{1}{f_E} = \frac{1}{(\alpha_E + \gamma) E_b^{-1}(A) (K)^{\alpha_K + \varepsilon} \exp \left(\delta_k K^{\frac{\alpha_K + \varepsilon}{\alpha_K}} \right) (E_b)^{\alpha_E + \gamma} (L)^{\alpha_L}} \quad (1)$$

Where $f_E(.)$ is the marginal product of energy and the subscript b in E denotes “boundary” level of E .

At the minimum (c_b, E_b) , $\frac{\partial V}{\partial K} (f(.) - c_b - \lambda K) + E_b = 0$. Hence:

$$E_b = \left(f(.) - \lambda K - r_b / \frac{\partial V}{\partial K} \right) \quad (2)$$

Plugging (1) in (2) yields:

$$E_b = \left[\frac{c_b + \lambda K}{1 - (\alpha_E + \gamma)} \right]^{\frac{1}{(\alpha_E + \gamma)}} K^{-\frac{(\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} \exp \left(- \frac{\delta_k}{(\alpha_E + \gamma)} K^{\frac{\alpha_K + \varepsilon}{\alpha_K}} \right) L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \quad (3)$$

Our viability kernel is depicted by:

$$V(c_b) = \int_K^\infty \frac{1}{f_E(A, A_E E_b(c_b), A_K K, A_L L)} dx + S_b \quad (4)$$

Plugging (3) in (1) and the resulting expression in (4) yields:

$$V(K, c_b, S_b) = A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \frac{1}{(\alpha_E + \gamma)} \left[\frac{c_b}{1 - (\alpha_E + \gamma)} \right]^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \int_K^\infty x^{-\frac{(\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} \exp \left(-\frac{\delta_K x^{\frac{\alpha_K + \varepsilon}{\alpha_K}}}{(\alpha_E + \gamma)} \right) dx + S_b. \quad \text{The}$$

expression $\int_K^\infty x^{-\frac{(\alpha_K + \varepsilon)}{(\alpha_E + \gamma)}} \exp \left(-\frac{\delta_K x^{\frac{\alpha_K + \varepsilon}{\alpha_K}}}{(\alpha_E + \gamma)} \right) dx$ can be denoted as $\int_K^\infty t^a \exp(-t) dx$ which is the

well known upper incomplete gamma function $\Gamma(a+1, F(K))$. In this case $t = \frac{\delta_K x^{\frac{\alpha_K + \varepsilon}{\alpha_K}}}{(\alpha_E + \gamma)}$

and $a = -\frac{\alpha_K + \varepsilon}{\alpha_E + \gamma}$. After the transformation the viability kernel can be expressed as:

$$V(K, c_b, S_b) = A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \frac{\alpha_K}{(\alpha_E + \gamma)(\alpha_K + \varepsilon)} \left[\frac{c_b}{1 - (\alpha_E + \gamma)} \right]^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \left(\frac{(\alpha_E + \gamma)}{\delta_K} \right)^{\frac{\alpha_K(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_K + \varepsilon)(\alpha_E + \gamma)}} \Gamma(a+1, F(K)) + S_b \quad (5)$$

The upper incomplete gamma function is finite if and only if $1 + a > 0$. This implies that

$\frac{(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{\alpha_E + \gamma} > 0$. In addition the kernel is positive and finite if and only if

$1 > (\alpha_E + \gamma)$ and $\delta_K > 0$ ■

Maximum Sustainable Consumption

The maximum level of sustainable consumption will be achieved if we allow the stock of the natural resource to converge to zero asymptotically. In addition to compute this level we need to find the c_b that corresponds to the existing stock of natural resource S_0 .

Therefore we set $V(K, c_b, S_b = 0) = S_0$ and the viability kernel can now be denoted as:

$$S_0 = A^{-\frac{1}{(\alpha_E + \gamma)}} L^{-\frac{\alpha_L}{(\alpha_E + \gamma)}} \frac{\alpha_K}{(\alpha_E + \gamma)(\alpha_K + \varepsilon)} \left[\frac{c_b}{1 - (\alpha_E + \gamma)} \right]^{\frac{1 - (\alpha_E + \gamma)}{(\alpha_E + \gamma)}} \left(\frac{(\alpha_E + \gamma)}{\delta_K} \right)^{\frac{\alpha_K(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_K + \varepsilon)(\alpha_E + \gamma)}} \Gamma(a+1, F(K)) \quad (6)$$

Solving for c_b in equation (6) yields:

$$c_b = [1 - (\alpha_E + \gamma)] A^{\frac{1}{1-(\alpha_E + \gamma)}} L^{\frac{\alpha_L}{1-(\alpha_E + \gamma)}} \left(\frac{\delta_K}{(\alpha_E + \gamma)} \right)^{\frac{\alpha_K(\alpha_E + \gamma) - (\alpha_K + \varepsilon)}{(\alpha_K + \varepsilon)[1-(\alpha_E + \gamma)]}} \left[\frac{S_0 (\alpha_E + \gamma)(\alpha_K + \varepsilon)}{\alpha_K} \right]^{\frac{(\alpha_E + \gamma)}{1-(\alpha_E + \gamma)}} \Gamma(a+1, F(K))^{\frac{(\alpha_E + \gamma)}{1-(\alpha_E + \gamma)}}$$

Appendix B: Programs for Econometric Estimation

Aggregation of inputs and output

Aggregate level of material or inputs (capital, labor, and energy) in period t is calculated as:

$$x_i^t = \exp \left[\sum_{k=1}^{35} \alpha_{ik}^t \ln(x_{ik}^t) \right]$$

Where x_i^t is the aggregate level of material or input i (capital, labor, and energy) used at time t, x_{ik}^t is the level of material or input i used in sector k at time t, and α_{ik}^t is the share of material or input i used in sector k on the total value of material or input i in t.

Cobb Douglas:

The equation to be estimated is:

$$\ln m = \alpha_K \ln K + \alpha_L \ln L + \alpha_E \ln E$$

A=[10.1977	11.6291	9.8724
10.2567	11.6484	9.8983
10.2514	11.6753	9.9387
10.2852	11.7056	9.9603
10.3421	11.7174	9.9882
10.3823	11.7446	10.0108
10.4316	11.7788	10.0673
10.5267	11.7849	10.1431
10.5970	11.8032	10.2113
10.7041	11.8275	10.2624
10.7927	11.8322	10.3083
10.8087	11.8389	10.3166
10.8463	11.8557	10.3393
10.9254	11.8946	10.3552
10.9785	11.8808	10.3871
10.9474	11.8662	10.3994
10.9317	11.8506	10.4520
10.9798	11.9209	10.4979
11.0675	11.9855	10.5007
11.1236	12.0450	10.5987
11.1956	12.0493	10.7132
11.2245	12.0832	10.6798
11.2994	12.0690	10.6558
11.3620	12.1644	10.5675

11.3794	12.2370	10.6100
11.4714	12.2862	10.5325
11.5576	12.3198	10.3046
11.5921	12.3855	10.3740
11.6541	12.4234	10.3407
11.6621	12.4856	10.3729
11.7287	12.5307	10.4561
11.8004	12.5226	10.3809
11.8520	12.5395	10.3680
11.8872	12.5895	10.3625
11.8904	12.6331	10.3462
11.9345	12.6777	10.3454
11.9617	12.7044	10.4195
12.0158	12.7438	10.3494
12.0923	12.7837	10.3519
12.1491	12.8151	10.4116
12.2222	12.8388	10.6280
12.2633	12.8587	10.4474
12.3246	12.8548	10.4454
12.3307	12.8903	10.4177
12.3385	12.9267	10.4629
12.3349	12.9441	10.5058];

m=[11.2372

11.2592
 11.3117
 11.3603
 11.4046
 11.4680
 11.5240
 11.5539
 11.6109
 11.6569
 11.6517
 11.6993
 11.7712
 11.8347
 11.8187
 11.7617
 11.8000
 11.8707
 11.9300
 11.9594
 11.9088
 11.9231
 11.9167

11.9450

12.0127

12.0621

12.1089

12.1710

12.2029

12.2143

12.2470

12.2405

12.2958

12.3364

12.3865

12.4374

12.5030

12.5564

12.6396

12.7007

12.7557

12.7518

12.7618

12.7877

12.8252

12.8557];

X1 = [A(:,1) A(:,2) A(:,3)];

regstats(m,X1,'linear')

Where m is materials, A(:,1) is natural log of capital, A(:,2) is natural log of labor, A(:,3), and is natural log of energy.

Transcendental:

The equation to be estimated is:

$$\ln m = \alpha_k \ln K + \alpha_L \ln L + \alpha_E \ln E + \delta_k K$$

B=[10.1977	11.6291	9.8724	26842
10.2567	11.6484	9.8983	28474
10.2514	11.6753	9.9387	28323
10.2852	11.7056	9.9603	29295
10.3421	11.7174	9.9882	31011
10.3823	11.7446	10.0108	32283
10.4316	11.7788	10.0673	33913
10.5267	11.7849	10.1431	37297
10.5970	11.8032	10.2113	40015
10.7041	11.8275	10.2624	44538
10.7927	11.8322	10.3083	48663
10.8087	11.8389	10.3166	49450
10.8463	11.8557	10.3393	51344
10.9254	11.8946	10.3552	55568

10.9785	11.8808	10.3871	58601
10.9474	11.8662	10.3994	56804
10.9317	11.8506	10.4520	55924
10.9798	11.9209	10.4979	58678
11.0675	11.9855	10.5007	64053
11.1236	12.0450	10.5987	67752
11.1956	12.0493	10.7132	72810
11.2245	12.0832	10.6798	74942
11.2994	12.0690	10.6558	80776
11.3620	12.1644	10.5675	85995
11.3794	12.2370	10.6100	87504
11.4714	12.2862	10.5325	95932
11.5576	12.3198	10.3046	104567
11.5921	12.3855	10.3740	108242
11.6541	12.4234	10.3407	115158
11.6621	12.4856	10.3729	116086
11.7287	12.5307	10.4561	124084
11.8004	12.5226	10.3809	133303
11.8520	12.5395	10.3680	140362
11.8872	12.5895	10.3625	145399
11.8904	12.6331	10.3462	145865
11.9345	12.6777	10.3454	152442
11.9617	12.7044	10.4195	156644
12.0158	12.7438	10.3494	165342
12.0923	12.7837	10.3519	178489
12.1491	12.8151	10.4116	188933
12.2222	12.8388	10.6280	203259
12.2633	12.8587	10.4474	211776
12.3246	12.8548	10.4454	225159
12.3307	12.8903	10.4177	226542
12.3385	12.9267	10.4629	228317
12.3349	12.9441	10.5058	227489];

m=[11.2372

11.2592

11.3117

11.3603

11.4046

11.4680

11.5240

11.5539

11.6109

11.6569

11.6517

11.6993

11.7712

```

11.8347
11.8187
11.7617
11.8000
11.8707
11.9300
11.9594
11.9088
11.9231
11.9167
11.9450
12.0127
12.0621
12.1089
12.1710
12.2029
12.2143
12.2470
12.2405
12.2958
12.3364
12.3865
12.4374
12.5030
12.5564
12.6396
12.7007
12.7557
12.7518
12.7618
12.7877
12.8252
12.8557];
X2 = [B(:,1) B(:,2) B(:,3) B(:,4)];
regstats(m,X2,'linear')

```

Where m is materials, A(:,1) is natural log of capital, A(:,2) is natural log of labor, A(:,3), is natural log of energy, and A(:,4) is capital.

Appendix C: Parameter Estimates

Parameter	Estimate	<i>Cobb Douglas</i> p-Value
α (intercept)	0.8593587	0.6501
α_K	0.3567364	0.0255
α_L	0.4809408	0.032
α_E	0.1219218	0.0251

<i>Transcendental</i>		
Parameter	Estimate	p-Value
α (intercept)	4.856676	0.0075
α_K	0.272387	0.0375
α_L	0.137028	0.4718
α_E	0.203681	0.023
δ_K	0.0000031	1.7E-4

Appendix D: Programs for Simulations

Stock of Fossil Energy

(1) Sources	(2) 2005 Units ^a	(3) btus per unit	(4) Total btus	(5) MMBTU =(4) / 1 million	(6) Shares =(5) / 1.2E+12
Coal (short tons)	1.9E+10	20355372	3.9E+17	3.9E+11	0.334
Oil (barrels)	2.2E+10	5800000	1.3E+17	1.3E+11	0.109
gas 1 (cubic feet)	2.0E+14	1020	2.1E+17	2.1E+11	0.180
gas 2 (cubic feet)	2.1E+14	1020	2.2E+17	2.2E+11	0.188
gas 3 (cubic feet)	1.9E+14	1020	1.9E+17	1.9E+11	0.163
gas 4 (cubic feet)	2.8E+13	1020	2.9E+16	2.9E+10	0.025
gas 5 (cubic feet)	3.4E+10	1020	3.5E+13	3.5E+07	0.000
Total			1.2E+18	1.2E+12	
Aggregate Index					2.3E+11 ^b

^a Data available from <http://www.eia.doe.gov/>.

^b Calculated as $y = \exp(\sum_i s_i \cdot x_i)$ where x_i are quantities for each source in column 5 and s_i are shares in last column.

TRANSCENDENTAL

```
*****
MAXIMUM SUSTAINABLE CONSUMPTION
*****
```

```
-----
A=1; So=2.28712E+11; K=227489; L=418361; E=36527; M=1; lnk=12.33; lnL=12.94;
lne=10.505; alphak=0.27238; alphas=0.137028; alphas=0.20368; alphas=0;
deltak=0.00000311; epsilon=0; gamma=0;
```

Sustainability rejected then imposed (alphae=0.3) and simulated:

```
*****
SUSTAINABLE CONSUMPTION AND HICKS-NEUTRAL TECHNICAL CHANGE
*****
```

```

-----
A=[1:0.0001:2.7182]; So=2.28712E+11; K=227489; L=418361; E=36527; lnk=12.33;
lnl=12.94; lne=10.505; alphak=0.27238; alphas=0.137028; alphae=0.3;
deltak=0.00000311; epsilon=0; gamma=0;
Cb=(A.^(1./(1-(alphae+gamma))))*(So.^((alphae+gamma)/(1-(alphae+gamma))))*((1-
(alphae+gamma)).^(1./(1-(alphae+gamma))))*((alphae+gamma).^(((alphae+gamma)-
(alphak+epsilon))./(alphae+gamma)))*((alphak+epsilon)./(alphak)).^((alphae+gamma)./(
1-(alphae+gamma)))*(deltak).^((alphak/(alphak+epsilon)).*(((alphae+gamma)-
(alphak+epsilon))./(1-(alphae+gamma))))*(L.^(alphas/(1-
(alphae+gamma))))*(gammainc(((deltak/(alphae+gamma)).*(K.^(alphak+epsilon)/alp
hak)), -alphak/(alphae+gamma)-epsilon/(alphak+epsilon)+1,'upper')).^(-
((alphae+gamma)/(1-(alphae+gamma)))));
x=log(A);
plot(x,Cb)

```

```

-----
*****
SUSTAINABLE CONSUMPTION AND NON-NEUTRAL TECHNICAL CHANGE
*****
Coefficients epsilon and gamma are not estimated but rather simulated based on
programs below

```

```

-----
A=1; So=2.28712E+11; K=227489; L=418361; E=36527; lnk=12.33; lnl=12.94;
lne=10.505; alphak=0.27238; alphas=0.137028; alphae=0.3; deltak=0.00000311;

[epsilon,gamma] = meshgrid(0:.00005:0.01,0:.00005:0.025);
*we assume epsilon and gamma start at zero and we simulate small positive changes
(technological progress) *
Cb=(A.^(1./(1-(alphae+gamma))))*(So.^((alphae+gamma)/(1-(alphae+gamma))))*((1-
(alphae+gamma)).^(1./(1-(alphae+gamma))))*((alphae+gamma).^(((alphae+gamma)-
(alphak+epsilon))./(alphae+gamma)))*((alphak+epsilon)./(alphak)).^((alphae+gamma)./(
1-(alphae+gamma)))*(deltak).^((alphak/(alphak+epsilon)).*(((alphae+gamma)-
(alphak+epsilon))./(1-(alphae+gamma))))*(L.^(alphas/(1-
(alphae+gamma))))*(gammainc(((deltak/(alphae+gamma)).*(K.^(alphak+epsilon)/alp
hak)), -alphak/(alphae+gamma)-epsilon/(alphak+epsilon)+1,'upper')).^(-
((alphae+gamma)/(1-(alphae+gamma)))));
mesh(epsilon,gamma,Cb)

```



```
*****
*****
ELASTICITY OF SUBSTITUTION AND NON-NEUTRAL TECHNICAL CHANGE
*****
*****
```

```
A=1; So=2.28712E+11; K=227489; L=418361; E=36527; M=1; lnk=12.33; lnL=12.94;
lne=10.505; alphak=0.27238; alphas=0.137028; alphae=0.3; alpham=0;
deltak=0.00000311;
[epsilon,gamma] = meshgrid(0:.0002:0.01,0:.0002:0.025);
y=A.*(L.^alpha).*(K.^(alphak+epsilon)).*(E.^(alphae+gamma)).*exp(deltak.*K.^((alph
ak+epsilon)./alphak));
F=(alphak+epsilon).*(K.^-1)+deltak.*((alphak+epsilon)./alphak).*(K.^(epsilon./alphak));
G=-(alphak+epsilon).*(K.^-
2)+deltak.*((alphak+epsilon)./alphak).*(epsilon./alphak).*(K.^(epsilon-
alphak)./alphak));
D=(alphae+gamma).*(E.^-1);

fk=F.*y;
fl=alphaL.*(L.^-1).*y;
fe=D.*y;
fkk=G.*y+(F.^2).*y;
fkl=F.*alphaL.*(L.^-1).*y;
fke=D.*F.*y;
fll=-alphaL.*(L.^-2).*y+alphaL.*(L.^-1).*alphaL.*(L.^-1).*y;
fle=alphaL.*(L.^-1).*D.*y;
fel=fle;
fee=-(alphae+gamma).*(E.^-2).*y+(alphae+gamma).*(E.^-1).*D.*y;
fek=fke;

Q1=-fk.*(fk.*fll.*fee+fkl.*fel.*fe+fek.*fle.*fl-fek.*fll.*fe-(fel.^2).*fk-fl.*fkl.*fee);
Q2=fkk.*(2.*fl.*fle.*fe-(fe.^2).*fll-(fl.^2).*fee);
Q3=-fkl.*(fl.*fek.*fe+fe.*fle.*fk-(fe.^2).*fkl-fee.*fk.*fl);
Q4=fke.*(fek.*(fl.^2)+fk.*fe.*fll-fe.*fl.*fkl-fk.*fl.*fel);
d=(Q1+Q2+Q3+Q4);

Q5=(fk.*fll.*fe+(fl.^2).*fke-fe.*fl.*fkl-fk.*fl.*fel);
dek=(Q5);
sigmake=((K.*fk+L.*fl+E.*fe)./(K.*E)).*(dek./d);

Q6=(2.*fk.*fkl.*fl-(fl.^2).*fkk-(fk.^2).*fll);

dee=(Q6);
sigmaee=((K.*fk+L.*fl+E.*fe)./(E.*E)).*(dee./d);
sigmaMke=((fe.*E)./(K.*fk+L.*fl+E.*fe)).*(sigmake-sigmaee);
mesh(epsilon,gamma,sigmaMke)
```

CHAPTER 2

ENVIRONMENTAL EFFICIENCY AMONG CORN ETHANOL PLANTS

1. Introduction

The U.S. corn ethanol industry has benefited from government support due to its potential to achieve a rather wide set of goals: mitigating emissions of greenhouse gases (GHG), achieving energy security (diversifying energy sources), improving farm incomes and fostering rural development among others. Continuation of policy support, however, is being debated due to doubts about the direct and indirect GHG effects of the industry. Moreover, the capacity of the industry to reduce GHG emissions per gallon of ethanol produced may also determine the opportunities opened to it in future carbon markets and in the National Renewable Fuel Standard program. This study provides information relevant to these issues by measuring the environmental performance of the industry in terms of GHG emissions and the economic cost (shadow price) of GHG reductions.

Input requirements and byproducts' yield per gallon of ethanol produced are critical in determining environmental performance. Previous studies have addressed the issue of input requirements and byproducts' yield of ethanol plants. Using engineering data McAloon et al. (2000) and Kwiatkowski et al. (2006) measured considerable improvement in plant efficiency between 2000 and 2006. Shapouri, et al. (2005) reported input requirements and cost data based on a USDA sponsored survey of plants for the year 2002. Wang et al. (2007) and Plevin et al. (2008), reported results based on spreadsheet models of the industry (GREET and BEACCON, respectively). Pimentel et al. (2005) and Eidman (2007) reported average performances of plants although they do

not clearly indicate the sources of their estimates. Finally Perrin et al. (2009) reported results on input requirements, operating costs, and operating revenues based on a survey of seven dry grind plants in the Midwest during 2006 and 2007.

With the exception of Shapouri et al. (2005) and Perrin et al. (2009) all of these studies reported values corresponding to the average plant rather than to individual plants. In addition, it is generally believed that the industry has become more efficient and technologically homogeneous since 2005. Since the data used in Shapouri et al. (2005) was collected in 2002 it may not be representative of current technologies in the industry. In contrast to Shapouri et al. (2005), Perrin et al. (2009) surveyed plants in operation during 2006 and 2007 and employed a much more restrictive sampling criterion (discussed below) which yielded a modern and technologically homogenous sample of plants. This sample is believed to be more representative of current technologies and is, hence, our data of choice to assess the environmental performance of plants. Based on these data the present study evaluates the environmental efficiency of seven recently constructed ethanol plants in the North Central region of the U.S. The returns over operating costs (ROOC)³⁰ that may be gained or lost by plants as a consequence of the effort to reach a given environmental target are also calculated and discussed.

2. Materials and Method

2.1. Data

The environmental performance of a plant is evaluated on the basis of emission of greenhouse gases associated with its productive activity. Greenhouse gas emissions from

³⁰ We evaluate economic performance based on returns over operating costs rather than profits. This is because capital costs are not included in our analysis.

plants were not directly measured but rather calculated based on observable inputs and outputs corresponding to each plant. In addition concerns regarding the environmental impact of ethanol production refer to life cycle³¹ GHG emissions and not only those emissions at the processing stage. Therefore we evaluate life cycle GHG emissions associated with observable inputs and outputs. Our observations consist of 33 quarterly reports of input and output quantities and prices from a sample of seven Midwest ethanol plants. Following the non parametric efficiency literature we refer to each observation as a decision making unit (DMU). Plants produce 3 outputs (ethanol, dry distillers grains with solubles (DDGS), and modified wet distillers grains with solubles (MWDGS)) using 7 inputs³² (corn, natural gas, electricity, labor, denaturant, chemicals, and “other processing costs”).

2.2. Ethanol Plants: Characteristics

Table 1 presents some quarterly characteristics of the seven dry grind ethanol plants surveyed. According to Table 1 the plants produced an average rate equivalent to 53.1 million gallons of ethanol per year, with a range from 42.5 million gallons per year to 88.1 million gallons per year. The period surveyed included the third quarter of 2006 until the fourth quarter of 2007 (six consecutive quarters). In addition plants could be differentiated by how much byproduct they sold as DDGS (10% moisture) compared to

³¹ “Life cycle” in this case includes emissions taking place at three stages of the production process: corn production (farmers), ethanol production (biorefinery), and feedlot (byproducts from ethanol plants are given a credit for replacing corn as feed in livestock production).

³² Results of our survey contained total expenditures in labor, denaturant, chemicals, and other processing costs. As a result we calculated implicit quantities for these inputs dividing total expenditures by their corresponding price indexes. Labor and management price index associated to the Basic Chemical Manufacturing Industries was obtained from http://www.bls.gov/oes/current/naics4_325100.htm#b00-0002. Denaturant, chemicals and other processing costs were calculated based on the Producer Input Price Index for “All other basic inorganic chemicals”, <http://www.bls.gov/pPI/>.

MWDGS (55% moisture). Variation on this variable was significant, averaging 54% of byproduct sold as DDGS, but ranging from one plant that sold absolutely no byproduct as DDGS to another plant that sold nearly all byproduct (97%) as DDGS.

Finally, Table 1 briefly characterizes plant marketing strategies. In purchasing input feedstock, five of the six plants purchased corn via customer contracts. Similarly, in selling ethanol, five of the six plants used third parties or agents. Byproduct marketing across plants displayed a higher degree of variance. Marketing of DDGS was split fairly evenly between spot markets and third parties/agents. An even higher variability was observed for MWDGS, where no one marketing strategy (spot market, customer contract, or third party/agent) was significantly more prevalent across plants than any other.

Table 2 displays descriptive statistics of inputs used and outputs produced by the 33 DMUs in our sample. As mentioned before the basic observations in this study corresponds to a plant in a given quarter; so two quarters of the same plant are considered as two different observations as are two plants in the same quarter.

2.3. Environmental Performance of Ethanol Plants

2.3.1. Emissions Measurement

No direct measurements of GHG emissions are available in this industry; however they can be calculated using engineering relationships. A number of computer packages have been developed to facilitate these calculations (Wang et al. 2007; Farrell et al. 2006). We used the Biofuels Energy Systems Simulator³³ (BESS). The BESS model includes all GHG emissions from the burning of fossil fuels used directly in crop

³³ BESS is a software developed by a team of specialists in the Agronomy Department at the University of Nebraska, Lincoln (Liska, et al, 2009a, 2009b, <http://www.bess.unl.edu/>)

production, grain transportation, biorefinery energy use, and coproduct transport. All upstream energy costs and associated GHG emissions with production of fossil fuels, fertilizer inputs, and electricity used in the production life cycle are also included. Since these calculations involve modeling of crop production and feedlot and these display regional differences, BESS includes regional scenarios and an average scenario for the whole Midwest region. Plants in our sample are scattered across the Midwest and, hence, we have used scenario 2 in BESS “US Midwest average UNL” which is deemed representative of the whole region.

The BESS calculations of GHG emissions associated with a dry mill plant are equivalent to the following linear relationship:

$$GHG_{Mg} = 0.00668274 x_c + 0.063015823 x_{NG} + 0.0007445 x_{elect} + 0.000316916 u_{Eth} - 0.4197522186 u_{DDGS} - 0.407868 u_{MWDGS} \quad (1)$$

Where GHG_{Mg} represents megagrams of life cycle CO2 equivalent greenhouse gases, x_c is bushels of corn used by the plant, u_{DDGS} and u_{MWDGS} are tons of byproduct sold as dried and modified wet respectively by the plant, x_{NG} is the total amount of natural gas used by the plant measured in MMBTUs, x_{elect} is total amount of kilowatt hours (kwh) of electricity used by the plant, and u_{Eth} is the plant’s ethanol production in gallons.

Eq. (1) states that a bushel of corn used in an ethanol plant is associated with about 0.0067 megagrams of GHG emitted during the production of that bushel used in the biorefinery. DDGS and MWDGS have a positive and a negative component. The former is due to additional energy used in reducing moisture.³⁴ The latter are “credits” attributed

³⁴ In particular MWDGS require the use of electricity to centrifuge the wet byproduct and DDGS require the use of natural gas for heating and drying the wet byproduct.

to byproducts (i.e. reductions in GHG) due to the replacement of corn that would have been fed to livestock had the byproduct not been sold. The coefficient for ethanol production represents the combination of emissions associated with depreciable capital (0.0002050) and freight for grain transportation (0.000111916), expressed on a per gallon basis.

Eq. (1) includes outputs $u^j = (u_{Eth}^j, u_{MWDGS}^j, u_{DDGS}^j)$ and a pollution increasing subset of all inputs used by ethanol plants³⁵ denoted by $x_p^j = (x_c^j, x_{NG}^j, x_{elect}^j)$, where subindex p indicates pollutant. We can now re express Eq. (1) in vector notation. To do so we partition inputs and outputs into a column vector of pollution increasing inputs and output $a^j = (x_c^j, x_{NG}^j, x_{elect}^j, u_{Eth}^j)'$ and a column vector of pollution reducing byproducts $u_b^j = (u_{MWDGS}^j, u_{DDGS}^j)'$. The level of greenhouse gas emissions associated with a particular plant j as a function of observable inputs and outputs can be expressed as:

$$GHG^j = \alpha a^j + \beta u_b^j \quad (2)$$

Where $\alpha = (0.0066, 0.0630, 0.00074, 0.000316)$ is the 1x4 row vector of coefficients associated with pollution increasing categories a^j , and $\beta = (-0.419752, -0.407868)$ is the 1x2 row vector of coefficients associated with pollution reducing byproducts u_b^j .

2.3.2. Characterization of Potential Ethanol Technology From Individual Plant Data

Plants are constrained by a technology transforming a vector of N inputs $x = (x_1, x_2, \dots, x_N) \in \mathfrak{R}_+^N$ into a vector of M outputs $u = (u_1, u_2, \dots, u_M) \in \mathfrak{R}_+^M$. Observed

³⁵ As described before ethanol plants use 7 inputs in production. However only three of them increase life-cycle emissions of GHGs: corn, natural gas, and electricity.

combinations of inputs used and outputs produced (x^j, u^j) are taken to be representative points from the feasible ethanol technology. In this study we use data envelopment analysis (DEA) to infer the boundaries of the feasible technology set from the observed points, following the notation in Färe, et al.

Observations from the technology consist of a sample of J DMUs producing M outputs and using N inputs. The production technology can be represented by a graph denoting the collection of all feasible input and output vectors:

$$GR = \left\{ (x, u) \in \mathfrak{R}_+^{N+M} : x \in L(u) \right\}$$

Where $L(u)$, is the input correspondence which is defined as the collection of all input vectors $x \in \mathfrak{R}_+^N$ that yield at least output vector $u \in \mathfrak{R}_+^M$.

The frontier of the graph GR and observed levels of inputs and outputs will serve as references for environmental efficiency assessment.

2.3.3. Environmental Efficiency Measurement

A given DMU is deemed more environmentally efficient whenever it chooses a feasible (subject to the graph) combination of inputs and byproducts (DDGS and MWDGS) that results in lower GHG emissions while maintaining its ethanol production level at the observed value denoted by $\overline{u_{Eth}^j}$. Fixing ethanol production to its observed level, and assuming variable returns to scale and strong disposability of inputs and outputs the graph can be denoted by:

$$GR^j(V, S, \overline{u_{Eth}^j}) = \left\{ (x, u) : u_b^j \leq zM_b, x^j \geq zN, zu_{Eth} = \overline{u_{Eth}^j}, \sum_{j=1}^{33} z^j = 1, j = 1, \dots, 33 \right\} \quad (3)$$

Where z depicts a row vector of 33 intensity variables, M_b is the 33x2 matrix of observed byproducts, u_b^j is the 1x2 vector of observed byproducts corresponding to the j th DMU, N is the 33x7 matrix of observed inputs, x^j is the 1x7 vector of observed inputs corresponding to the j th DMU, u_{Eth} is the 33x1 vector of observed outputs, and $\overline{u_{Eth}^j}$ is the observed ethanol production by observation j .

We define the set of all combinations of corn, gas, electricity and byproducts that result in lower emissions than those actually produced by the j th DMU as:

$$GHG_g^j(x_p^j, u_b^j, \overline{u_{Eth}^j}) = \left\{ (x_p^{j'}, u_b^{j'}) : \alpha_x x_p^{j'} + \beta u_b^{j'} \leq \alpha_x x_p^j + \beta u_b^j \right\} \quad (4)$$

Where α_x is a subset of the vector α previously defined which does not include the coefficient for ethanol, i.e. $\alpha_x = (0.006682, 0.063015, 0.000744)$ and the rest is as before.³⁶

From Eq. (4) we can derive an isopollution line in DDGS and corn space, i.e. combinations of DDGS and corn that result in the same level of emissions keeping everything else constant. Fig. 1 depicts this set graphically in the corn and DDGS space (i.e. keeping everything else in the GHG equation fixed). The set GHG_g^j consists of all those points above the isopollution line as indicated by the arrows with direction northwest.

³⁶ We denote the coefficient associated with ethanol by $\gamma = 0.000316$. Ethanol production and its associated coefficient are included in both sets. However, since ethanol is fixed at the observed level $\overline{u_{Eth}^j}$, the complete version of the inequality is $\alpha_x x_p^{j'} + \beta u_b^{j'} + \gamma \overline{u_{Eth}^j} \leq \alpha_x x_p^j + \beta u_b^j + \gamma \overline{u_{Eth}^j}$ which after elimination is equivalent to the expression in (4).

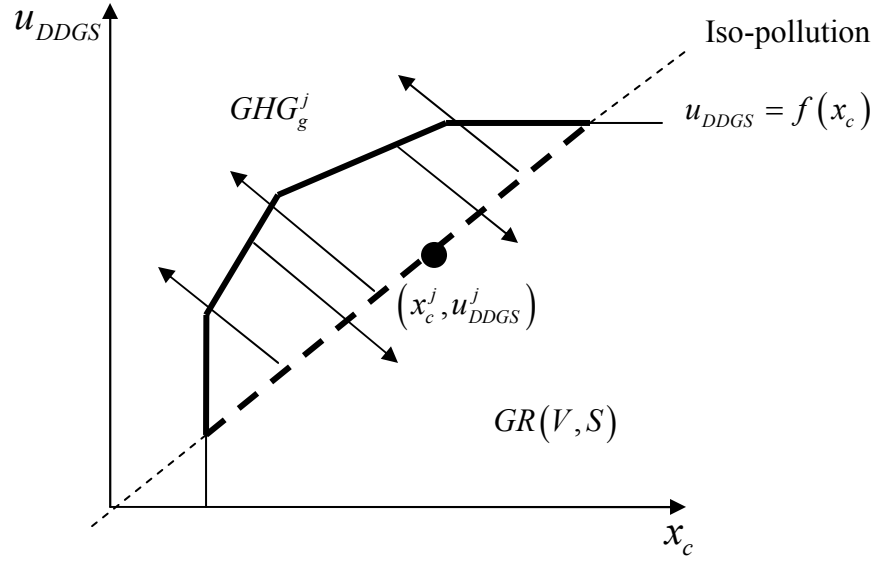


Figure 1. Iso-pollution and Sets

In Fig. 1 the feasible technology set is represented by a graph displaying variable returns to scale and strong disposability of inputs and outputs as indicated by the arrows moving from the frontier ($u_{DDGS} = f(x_c)$) with direction southeast. As clearly seen in Fig. 1, the set GHG_g^j includes combinations outside the graph and hence not attainable by DMUs in the sample. The subset of observations in GHG_g^j that belong to the graph and are hence attainable by DMUs is depicted by the intersection of both sets delimited by the bold lines in Fig. 1:

$$GHG_g^j(x_p^j, u_b^j, \overline{u_{Eth}^j}) \cap GR(V, S, \overline{u_{Eth}^j}) \quad (5)$$

The j th DMU could choose any alternative production plan within the area denoted by the bold lines to produce its ethanol production level, achieving a reduction in emissions while simultaneously increasing DDGS or reducing corn or both. In this study, the environmental **technically** efficient projection of a given observation to the boundary of the technology set follows a hyperbolic path defined by equiproportional reductions in

inputs and increases in byproducts. The value of the proportionate change necessary to encounter the boundary, ETE_g^j , is defined as the technical environmental efficiency of plant j:

$$ETE_g^j(x_p^j, u_b^j, \overline{u_{Eth}^j}) = \min \left\{ \lambda : GHG_g(\lambda x_p^j, \lambda^{-1} u_b^j) \cap GR(V, S, \overline{u_{Eth}^j}) \neq \emptyset \right\} \quad (6)$$

Where λ is a scalar defining the proportionate changes and the rest is as before. We calculated the value of $ETE_g^j(x_p^j, u_b^j, \overline{u_{Eth}^j})$ using MATLAB as indicated in Appendix A.

Environmental technical efficiency defined in Eq. (6) is illustrated in Fig. 2 by the distance from (x_c^j, u_{DDGS}^j) to point A which corresponds to the environmental technically efficient allocation in corn and DDGS space.

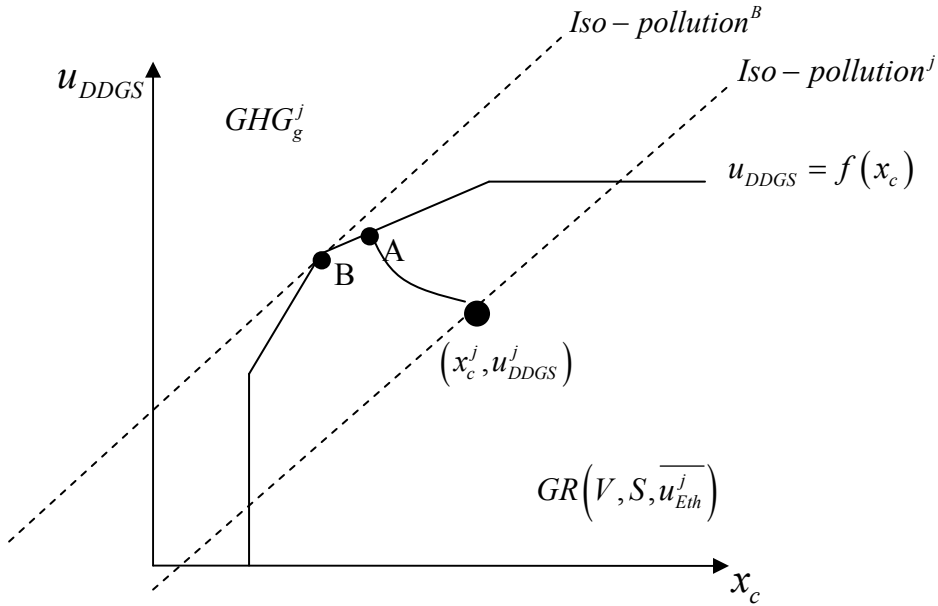


Figure 2. Environmental Technical Efficiency

Note however that point A does not correspond to the minimum feasible GHG level since it does not coincide with the point of tangency between the isopollution and the

graph (point B). The allocation that achieves the minimum level of GHG emissions subject to the graph is called the **overall** environmental efficient allocation.

Technically, we define this minimum feasible level of GHG emissions as:

$$\underline{GHG}^j(\overline{u_{Eth}^j}) = \min_{x_p, u_b} \left\{ GHG = \alpha_x x_p + \beta u_b + \gamma \overline{u_{Eth}^j} \quad s.t. \quad (x_p, u_b) \in GR(V, S, \overline{u_{Eth}^j}) \right\} \quad (7)$$

Where $\underline{GHG}^j(\overline{u_{Eth}^j})$ denotes minimum emissions attainable by j subject to observed ethanol production $\overline{u_{Eth}^j}$, x_p is the vector of pollution increasing inputs, u_b is the vector of byproducts and the rest is as defined before. The empirical calculation of Eq. (7) is described in Appendix B.

Overall environmental efficiency, E_g^j , is measured by the hyperbolic distance between a given observation j and the isopollution line corresponding to $\underline{GHG}^j(\overline{u_{Eth}^j})$. The hyperbolic distance is computed through calculation of the reduction of observed inputs and equiproportional expansion of observed byproducts such that the isopollution corresponding to $\underline{GHG}^j(\overline{u_{Eth}^j})$ is reached. This is illustrated by Fig. 3 where overall environmental efficiency is the distance between (x_c^j, u_{DDGS}^j) and point C.

The hyperbolic movement from (x_c^j, u_{DDGS}^j) to C results from the following technical relationship.

PROPOSITION. The measure of overall environmental efficiency, E_g^j , is related to minimum GHG in the following manner:

$$\underline{GHG}^j = E_g^j \alpha x_p^j + (E_g^j)^{-1} \beta b^j \quad j = 1, 2, \dots, J \quad (8)$$

See Proof in Appendix C.

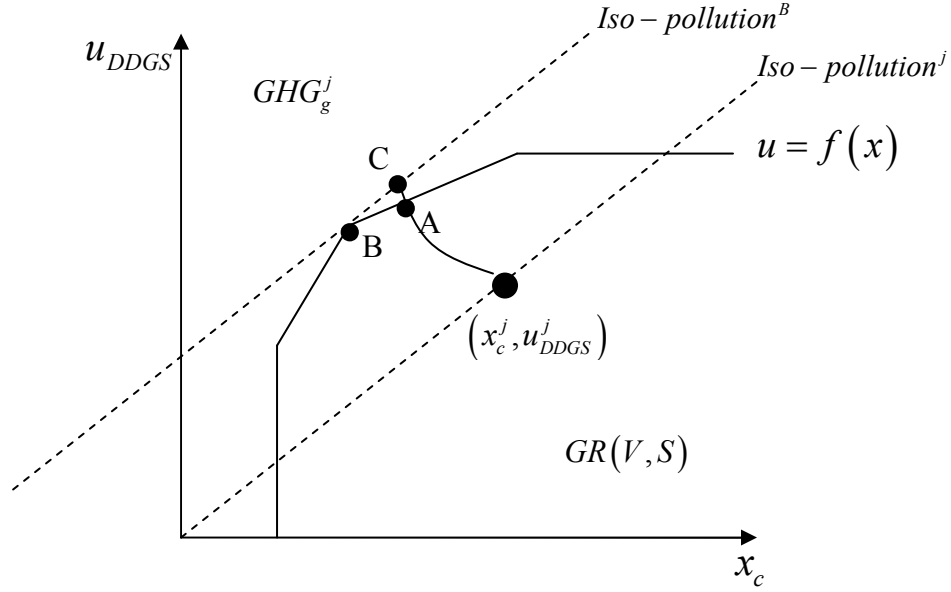


Figure 3. Decomposition of Overall Environmental Efficiency

We can decompose E_g^j into purely technical environmental efficiency ETE_g^j (represented graphically by the distance between (x_c^j, u_{DDGS}^j) and A) and environmental allocative inefficiency EAE^j (represented graphically by the distance between A and C). Overall environmental efficiency can be expressed as:

$$E_g^j = EAE_g^j ETE_g^j \quad (9)$$

Therefore, we can define allocative environmental inefficiency residually as:³⁷

$$EAE^j = \frac{E_g^j}{ETE_g^j} \quad (10)$$

Based on the solution to the problem described in Eq. (7) we calculate overall environmental efficiency by solving the implicit Eq. (8) for each observation. These

³⁷ Environmental allocative inefficiency was illustrated in Fig. 2 by the distance between the iso-pollution corresponding to combination A and iso-pollution corresponding to point D .

measures of environmental efficiency and their decomposition, Eq. (10), are calculated for our sample of surveyed dry grind ethanol plants and reported in Table 3. The minimum feasible GHG for each DMU as defined by Eq. (7) is calculated fixing ethanol production at observed levels.

2.4. ROOC and Environmental Targets: Trade off or Complementarity?

From Eq. (2) there is a clear relationship between GHG and the combination of inputs and byproducts. But there is also a relationship between combinations of inputs and byproducts and the level of ROOC. Therefore, in general, a change in GHG levels through reallocation of inputs and byproducts would bring about a change in ROOC. For a given level of ethanol production, the shadow price of GHG mitigation is the change in ROOC per unit change in GHG levels. The change in ROOC denotes the plant's maximum willingness to pay (WTP) for a permit to emit GHG. We define the shadow price of a ton of GHG as:

$$SV_{GHG}^j = \frac{WTP}{GHG_1^j - GHG_0^j} = \frac{\pi_1^j - \pi_0^j}{GHG_1^j - GHG_0^j} \quad (11)$$

Where WTP is willingness to pay for changing emissions from GHG_0^j to GHG_1^j . GHG_0^j denotes the original level of GHG and π_0^j the corresponding level of ROOC. GHG_1^j is the “targeted” level of GHG and π_1^j denotes ROOC at this targeted level. GHG level will be targeted at the minimum GHG (i.e. $GHG_1^j = \underline{GHG}^j$), or alternatively at the level corresponding to maximum achievable ROOC by firm j, π_*^j , which we designate as GHG_*^j .

2.4.1. Shadow Cost from Observed to ROOC Maximizing Allocation

We define the ROOC maximizing combination of inputs and byproducts (subject to a given level of ethanol production to make it comparable with the GHG minimizing combination) as the allocation that solves the following problem:

$$\pi_*^j(r^j, p^j, r_{Eth}^j, GR(V, S, \overline{u_{Eth}^j})) = \underset{x, u_b}{Max} \left\{ r_{Eth}^j \overline{u_{Eth}^j} + r^j u_b - p^j x \right\} \quad s.t. (u_b, x) \in GR(V, S, \overline{u_{Eth}^j}) \quad (12)$$

Where r_{Eth}^j is the observed price of ethanol obtained by observation j , $\overline{u_{Eth}^j}$ is the observed level of ethanol production by j , u_b is the 2x1 column vector of variable outputs (DDGS and MWDGS), r^j represents the 1x2 vector of observed prices of variable outputs (byproducts)³⁸ obtained by observation j , x is the 1x7 vector of variable inputs (corn, natural gas, electricity, labor, denaturant, chemicals, and “other processing costs”), and p^j represents the 1x7 vector of observed prices of variable inputs paid by j . Quantities of labor, denaturant, chemicals and others needed to calculate GR are obtained implicitly dividing total expenditures in these categories by their price indexes described in footnote 2. Prices for these categories in equation (12) are also those in footnote 2. We will denote the allocation that solves Eq. (12) with ethanol fixed at the observed level by $\{(x_*^j, u_*^j)\}$. The level GHG_*^j is calculated by inserting these values into (2).

We define the shadow value of GHG emissions associated with moving from the observed allocation to the ROOC maximizing allocation as:

³⁸ Three DMUs in our sample did not sell dried byproducts (they sold 100% MWDGS). Since we did not have reported DDGS prices for those three observations to calculate maximum ROOC we used average prices of DDGS obtained by other DMUs in the same quarter.

$$SV_{GHG}^j = \frac{\pi_*^j - \pi^j}{GHG_*^j - GHG^j} \quad (13)$$

An alternative shadow cost to Eq. (13) is that which is incurred by moving from the observed to the GHG minimizing combination of inputs and byproducts.

2.4.2. Shadow Cost from Observed to GHG Minimizing Allocation

The GHG minimizing combination is computed by solving Eq. (7) with ethanol production fixed at observed levels and minimum GHG denoted by \underline{GHG}^j . ROOC associated with this allocation (calculated by multiplying the GHG minimizing inputs and outputs times their respective prices) is designated as $\underline{\pi}^j$.

We define the shadow value of GHG related to a change from the observed to the GHG minimizing point as:

$$SV_{GHG}^j = \frac{\pi^j - \underline{\pi}^j}{\underline{GHG}^j - GHG^j} \quad (14)$$

Finally we consider the shadow value of GHG related to a change from the GHG minimizing to the ROOC maximizing point.

2.4.3. Shadow Cost from GHG Minimizing to ROOC Maximizing Allocation

Such a change is illustrated in Fig. 4 in the corn and DDGS space. In Fig. 4 the GHG minimizing combination is represented by point B (the isopollution line is denoted by \underline{GHG}^j). If relative prices are those corresponding to the slope of π_*^j then ROOC maximization is achieved at point A and this requires a decrease in corn and DDGS with

respect to the GHG minimizing point. ROOC at A are denoted by π_*^j and ROOC at B are π^j . Emissions at B are denoted by \underline{GHG}^j and emissions at A are $\overline{GHG}^j > \underline{GHG}^j$.

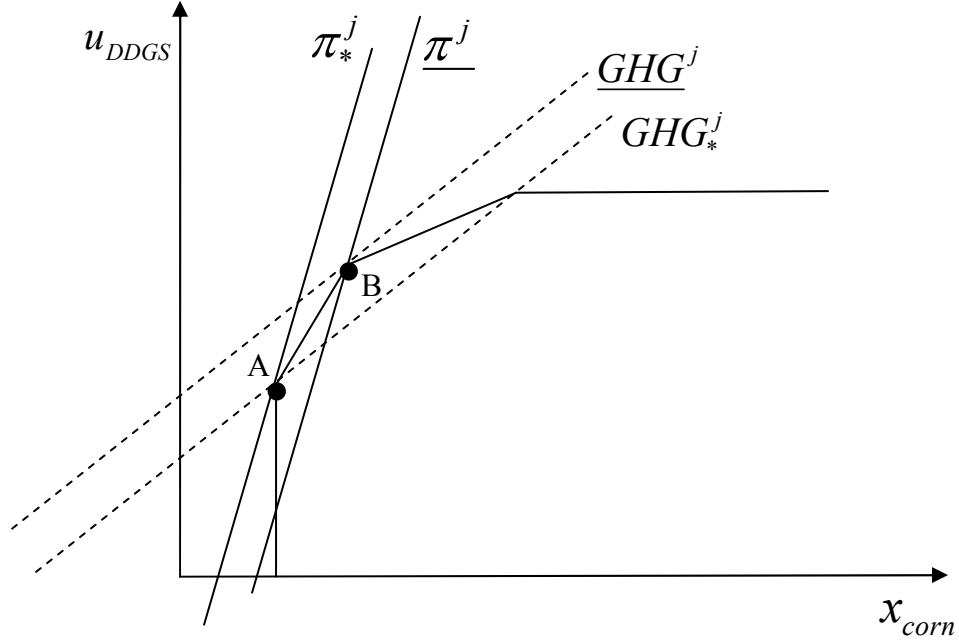


Figure 4. Shadow Cost from GHG Minimizing to ROOC Maximizing Allocation

The shadow value associated with a change from the GHG minimizing combination to the ROOC maximizing one is defined by:

$$SV_{GHG}^j = \frac{\pi_*^j - \pi^j}{\overline{GHG}^j - \underline{GHG}^j} \quad (15)$$

3. Results and Discussion

3.1. Environmental Performance of Ethanol Plants

Fixing ethanol production at observed levels, measures of environmental efficiency and their decomposition are calculated for our sample of surveyed dry grind ethanol plants and reported in Table 3. Results reveal that DMUs are very efficient from a

technical point of view and that most environmental inefficiency comes from allocative sources. Therefore DMUs seem to have room for GHG reductions mainly by changing input and output combinations subject to the graph. In particular, the average DMU may be able to reduce emissions by 6% which amounts to 3,116 tons of CO₂ equivalent GHGs per quarter (or 0.46 pounds per gallon of ethanol produced). The average DMU in our sample, at observed allocations, displays a GHG intensity of about 46 gCO₂e/MJ. At the GHG minimizing allocation, the average DMU in our sample displays a GHG intensity of 43 gCO₂e/MJ which is 6.5% lower than observed levels. This intensity is, for example, 55% lower than the target standard established by California by 2019 (86.27 gCO₂e/MJ). It is of interest to know what reallocations of inputs and byproducts may actually achieve this improvement and we will go back to this point in detail later.

3.2. ROOC and Environmental Targets

Shadow costs associated with moving from observed to ROOC maximizing allocations are reported in Table 4. Given the rather large variability across observations both the median and the average are reported as measures of central tendency. Table 4 displays some observations that are unusually high and others unusually low. These disproportionate deviations from the average are due to changes in inputs that affect ROOC but do not affect emissions. These inputs are labor, denaturant, chemicals, and other processing costs. We classify as “outlier” any observation whose value exceeds the average by more than 3 times the standard deviation.

An important conclusion we can extract from Table 4 is the fact that almost all DMUs reduce GHG emissions by moving from observed to maximum ROOC. This suggests

that, under our convexity assumptions, most DMUs (including the average DMU) may be able to increase ROOC and reduce GHG *simultaneously* which would in turn imply that these DMUs face no trade off between economic and environmental goals at current combinations of inputs and byproducts.

There are many potential reasons for the failure of DMUs to attain the ROOC-maximizing allocation. First plants may not face market conditions that allow them to reallocate byproducts from dry to wet or viceversa. A rather significant livestock production relatively near the plant has to be in place for DMUs to be able to sell a significant portion of their byproduct as wet. These market constraints are not captured by our analysis. Second the graph is assumed to be convex in our calculations. This may not represent technology accurately. There may be indivisibilities in the construction and later modifications (expansions or contractions) of plants that result in non-convexities of the graph. These non-convexities would prevent plants from choosing the ROOC-maximizing allocation depicted by the convex graph, rendering economic inefficiencies.

The fact that DMUs can rearrange inputs and byproducts in such a way that they can both increase ROOC and reduce emissions prompts the following questions:

1. What inputs are reduced or increased and which byproduct is reduced or increased in such a rearrangement?
2. Why are plants not exploiting these reallocations that achieve greater ROOC?

The answer to the first question for the average plant is provided in Table 5. The average DMU would achieve greater ROOC and lower GHG simultaneously mainly by reducing the use of corn, natural gas, and electricity per gallon of ethanol produced, reducing the production of MWDGS, and increasing production of DDGS. A part of

these reductions is achieved through elimination of inefficiencies that would take the DMUs to the technological frontier but for the most part they are achieved through rearrangements along the surface described by the boundary of the graph defined in Eq. (3)

The answer to the second question is not as straightforward. As noted in the discussion of the first question our DMUs may be able to increase ROOC and reduce GHG mainly by reducing corn, natural gas, and electricity per gallon of ethanol produced and per ton of DDGS produced.³⁹ The apparent engineering (in)ability to maximize ethanol and DDGS yields when compared to other DMUs in the sample seems to drive the difference between observed production plans and ROOC maximizing plans for many DMUs. A note of caution is in place here. These results are based on the assumption that all DMUs are constrained by the same technological frontier. Under the assumption of homogeneous technology any difference in performance is attributed to efficiency differences rather than to technological differences. However technological heterogeneities may be present and prevent some DMUs from achieving the performance of others in the sample.

Shadow costs associated with moving from observed to GHG minimizing allocations, Eq. (14), for each DMU, average, and median are reported in Table 6. Nine DMUs lose ROOC while reducing GHGs, thus facing positive shadow values of GHGs, meaning a cost. Seventeen DMUs increase ROOC while reallocating to the minimum GHG level. The fact that the average willingness to pay for a change in allocation ($\pi_E^j - \pi^j$) is positive while average change in GHG is negative, results in negative average shadow

³⁹ Reductions in MWDGS may come as a surprise. However given relative prices it appears as if this was a convenient reallocation for many DMUs.

values. Table 6 indicates that the average DMU may be able to increase ROOC while reducing GHG which again seems to suggest complementarity between goals. In particular the average DMU may be able to increase ROOC by about \$39 per ton of GHG reduced. The seventeen firms with negative shadow prices would presumably be willing to sell permits at any small price, since there is no ROOC lost from reducing their own GHGs.

Since there seems to be a great deal of variability in shadow prices of GHG across DMUs we have plotted a histogram that shows the approximate distribution of these values in Fig. 5.

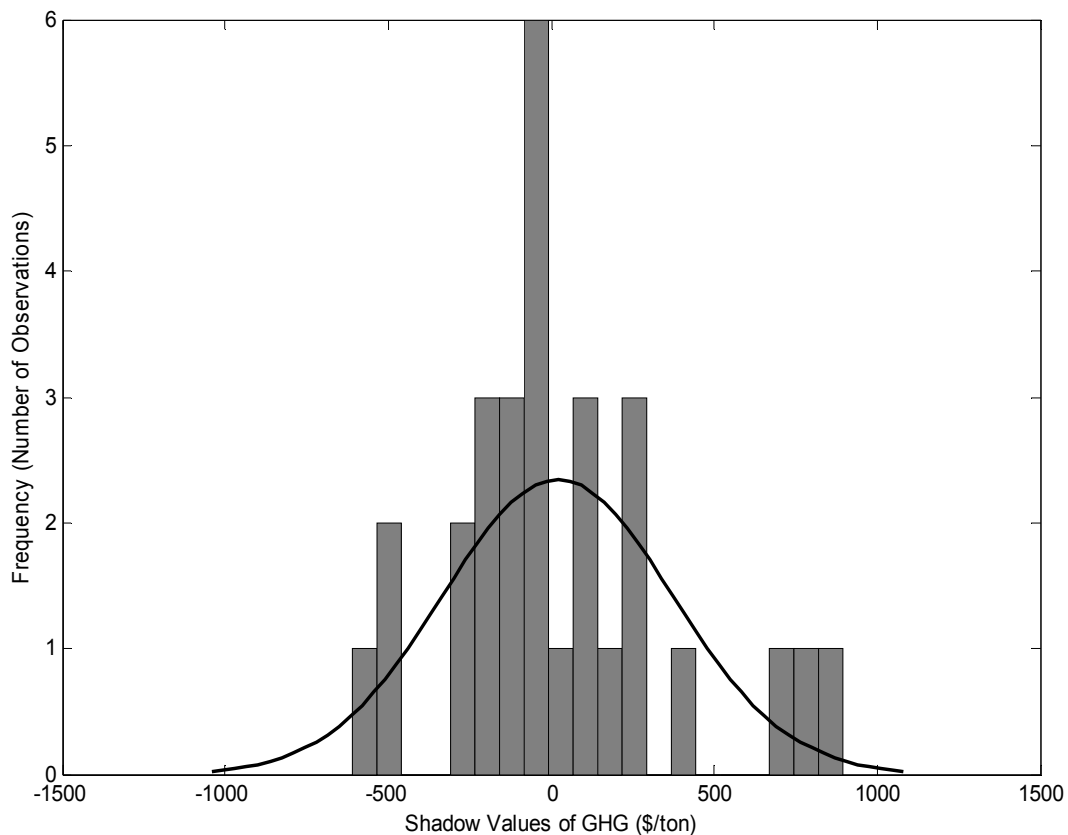


Figure 5. Histogram of Shadow Values (observed to GHG-minimizing)

The histogram does not take into account those observations deemed as outliers. The presence of outliers is mainly due, as discussed above, to changes in inputs affecting ROOC but not GHG, i.e. labor, denaturant, chemicals, and other processing costs. We have superimposed to the histogram a normal density function that smoothes out the distribution. Despite the variability across DMUs, the highest frequency of shadow values (i.e. most of the “mass” of the distribution) appears to be located around zero. This means that plants are approximately efficient in the sense that they are operating at levels for which the marginal value of GHG is around zero which is, in turn, the current GHG price that DMUs face.

According to Table 7 the average DMU achieves minimization of GHG through substantial reductions in DDGS and MWDGS which in turn allows it to significantly reduce natural gas and electricity. Finally reductions in corn per gallon of ethanol are also involved in this GHG minimization. Such reallocations not only achieve reductions in GHG but also increase ROOC (negative shadow value)

Shadow costs associated with moving from GHG minimizing to ROOC maximizing allocations, Eq. (15), for each DMU, average and median are reported in Table 8. All DMUs increase both ROOC and GHGs in moving from low GHG solution to high ROOC solution. The average DMU would forfeit \$1,726 in ROOC for each ton of GHG reduced, a very high cost of regulation if that firm were required to reduce GHGs. If DMUs are forced to reduce GHG emissions below ROOC maximizing levels, these shadow values indicate that they would be willing to purchase permits if the market value is in the vicinity of \$20 to \$30 per ton, rather than reduce one ton of GHG emissions. The histogram (with superimposed normal density) corresponding to Table 8 is plotted in Fig.

6. This histogram as the one in Fig. 5 does not take into account those observations classified as outliers.

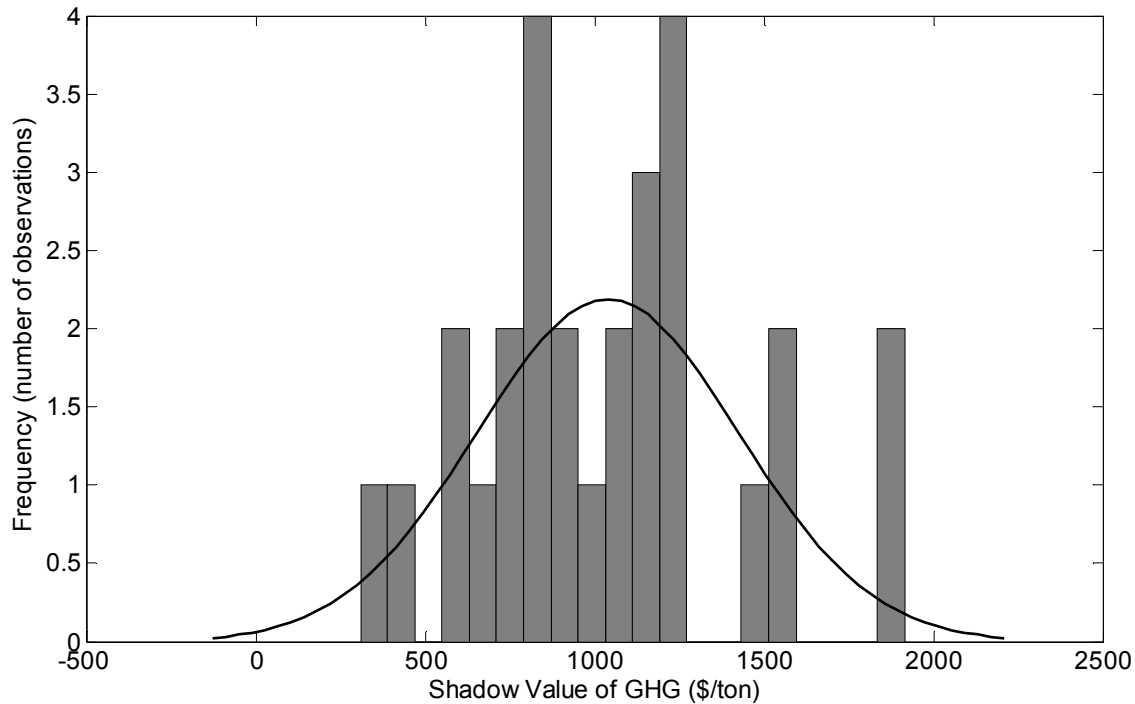


Figure 6. Histogram of Shadow Values (GHG Min. to ROOC Maximizing)

The reallocation of inputs and byproducts that would take the average DMU from the GHG minimizing to the ROOC maximizing combination is displayed in Table 9. The average DMU achieves increases in ROOC mainly through substantial increases in DDGS which in turn entails increases in natural gas and electricity, and reductions in MWDGS. Another very important component of ROOC increases is reductions of corn per gallon of ethanol produced.

Results for the average DMU in Tables 4, 6, and 8 can be combined to recover the shape of the relationship between GHG and ROOC. Plotting the three averages in the GHG and ROOC space yields the graph in Fig. 7. We denote the observed combination

of the average by (GHG^j, π^j) , the ROOC maximizing combination by (GHG_*^j, π_*^j) , and the GHG minimizing combination by $(\underline{GHG}^j, \underline{\pi}^j)$. There seems to be room for simultaneous improvement of environmental and economic performance, as previously indicated in discussions of Tables 4 and 6. However, if the average firm were able to adjust inputs and byproducts to the ROOC maximizing combination, it would face an intense trade off described just above.

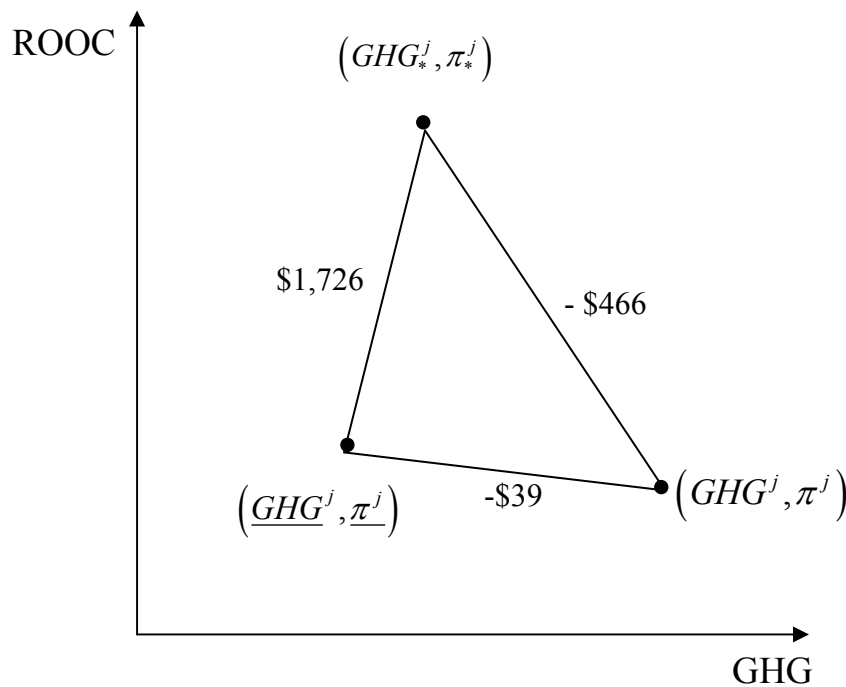


Figure 7. ROOC and GHG

4. Conclusions

The purpose of this study was to contribute to the ongoing debate regarding the merits and potential of the ethanol industry in the US by investigating the current environmental performance at the individual plant level, the potential for improvement in this performance and its effects on the industry's overall emissions of greenhouse gases.

Several important conclusions can be drawn from this study. First, our results suggest that decision making units (DMUs) may have some room for improving environmental performance. However since plants are technically very efficient, most of this improvement has to come from changes in combinations of inputs and byproducts along the frontier (reduction in environmental allocative inefficiencies). By eliminating allocative inefficiencies the average DMU could apparently decrease emissions by 6%, which amounts to about 3,116 tons of CO₂ equivalent GHG.

Negative shadow values of GHG from observed to ROOC maximizing combinations reveal that at current operating levels DMUs may be able to increase ROOC and reduce GHG *simultaneously*. This result points towards the conclusion that firms are not allocating inputs and outputs so as to maximize profits and, therefore, they may increase their economic viability helping their case for public support. However, once DMUs have maximized profits, our results suggest that they may face significant ROOC losses if they are forced to reduce GHG beyond that point. In this case the average DMU in this sample would be willing to pay up to \$1,726 for a permit to emit ton of GHG, rather than suffer the ROOC reduction revealed by the shadow price of reducing carbon from ROOC maximizing to GHG minimizing levels.

The measurement of corn ethanol plants environmental performance, their potential for improvement, and ROOC/emissions trade offs conducted in this study should inform the debate on whether there is a place for corn ethanol as a “clean” substitute for gasoline. In particular our results suggest that ethanol plants in our sample can produce energy with considerable lower (52% lower) GHG intensity than gasoline. Moreover these plants have some room for reducing this footprint even more by reallocating inputs

and byproducts. Such reallocations would achieve a 6.5% reduction in GHG rendering energy with a GHG intensity 55% lower than gasoline. In turn these reductions may be achieved at a moderate or none economic cost as strongly suggested by a negative shadow price of \$39 per gallon.

5. References

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Table 1. Characteristics of the seven surveyed plants

States Represented	Iowa, Michigan, Minnesota, Missouri, Nebraska, S. Dakota, Wisconsin				
Annual Production Rate (m. gal/y)	Smallest	42.5			
	Average	53.1			
	Largest	88.1			
Number of Survey Responses by Quarters	03_2006	5			
	04_2006	6			
	01_2007	7			
	02_2007	7			
	03_2007	7			
	04_2007	2			
Percent of Byproduct Sold as Dry DGS	Smallest	0			
	Average	54			
	Largest	97			
Primary Market Technique		Corn	Ethanol	DDGS	MWDGS
	Spot	0	0	3	1
	Customer Contract	5	1	0	1
	Third Party/Agent	0	5	2	2

Table 2. Descriptive Statistics: Inputs and Outputs

	Corn (million bushels)	Natural Gas (thousand MMBTUs)	Electricity (million kwh)	Ethanol (million gallons)	DDGS (thousand tons)	MWDGS (thousand tons)
Average	4.8	361	7.8	13.7	21.3	14.5
Std Dev	0.9	61	1.5	2.8	10	15.4
Min	3.6	297	6.7	10.6	0	199
Max	8	569	13.3	22.9	34.2	56.2

Table 3. Environmental Efficiency Decomposition

DMU	Technical Environmental Efficiency	Allocative Environmental Efficiency	Overall Environmental Efficiency	Reduction of GHG (tons) ^[a]	Reduction of GHG (%) ^[b]
1	0.977	1	0.961	3,268	6
2	1	0.931	0.931	6,227	11
3	0.985	0.970	0.956	3,617	7
4	1	0.951	0.951	3,801	7
5	1	1	0.993	567	1
6	0.979	1	0.973	2,331	4
7	1	0.948	0.948	4,697	9
8	1	0.947	0.947	4,704	8
9	1	1	1	0	0
10	1	0.959	0.956	3,539	7
11	1	0.989	0.989	950	2
12	1	1	1	0	0
13	1	0.940	0.940	8,007	9
14	1	0.949	0.949	4,625	9
15	1	0.944	0.944	4,804	9
16	1	0.974	0.974	2,015	4
17	1	0.985	0.985	1,098	2
18	1	0.938	0.938	5,178	10
19	1	0.987	0.987	1,133	2
20	1	1	1	0	0
21	1	0.947	0.947	4,611	9
22	1	0.967	0.967	2,736	5
23	1	0.974	0.974	2,023	4
24	1	0.985	0.985	1,199	2
25	1	0.970	0.970	2,614	5
26	1	1	1	0	0
27	1	0.917	0.917	7,941	14
28	1	0.956	0.956	3,708	7
29	1	0.961	0.961	3,068	6
30	1	0.964	0.964	2,831	6
31	0.993	0.980	0.973	2,239	4
32	1	0.992	0.992	684	1
33	1	0.914	0.914	8,662	14
Average	0.998	0.967	0.965	3,116	6

^[a] This is calculated by taking the difference between observed and minimum GHG emissions.

^[b] Reduction in GHG emissions from previous column as a percentage of observed emissions.

Table 4. Shadow Values of GHG: observed to ROOC maximizing combination

DMU	WTP for change in allocation, $\pi_*^j - \pi^j$ (\$)	Change in GHG emissions, $GHG_*^j - GHG^j$ (tons)	Shadow Value of GHG (\$/ton)
1	948,565	-2,618	-362
2	1,483,022	-5,648	-263
3	2,094,972	-2,728	-768
4	1,223,985	-3,105	-394
5	619,562	120	5,147 - outlier
6	1,263,224	-1,920	-658
7	1,515,535	-4,100	-370
8	2,398,535	-4,405	-545
9	3,199	0	INFINITE
10	850,101	-2,636	-322
11	719,229	-264	-2,726
12	1,382	0	INFINITE
13	2,175,472	-7,709	-282
14	1,597,466	-4,026	-397
15	1,751,089	-4,339	-404
16	825,632	-1,027	-804
17	1,692	0	INFINITE
18	1,540,254	-4,555	-338
19	1,230,951	-488	-2,521
20	258,318	295	877
21	1,797,859	-3,726	-483
22	1,975,711	-2,035	-971
23	781,594	-344	-2,269
24	1,041,712	-332	-3,141
25	2,192,398	-1,990	-1,101
26	9,613	0	INFINITE
27	2,301,210	-7,495	-307
28	1,252,438	-3,075	-407
29	1,439,841	-2,291	-629
30	1,106,262	-1,801	-614
31	727,808	-1,367	-532
32	1,396,934	271	5,154 - outlier
33	1,865,307	-8,663	-215
Mean	1,420,685	-3,052	-466
Median	1,439,841	-2,636	-546

Table 5. Reallocation from observed to ROOC maximizing combination

Category \ Measure	Corn	Natural Gas	Electricity	Dry	Wet
Average Change (%)	-5.88	-3.83	-0.41	26.03	-10.23

Table 6. Shadow Values of GHG: observed to GHG minimizing combination

DMU	WTP for change in allocation, $\pi_E^j - \pi^j$ (\$)	Change in GHG emissions, $GHG_E^j - GHG^j$ (tons)	Shadow Value of GHG (\$/ton)
1	659,193	-3,268	-202
2	443,897	-6,227	-71
3	134,209	-3,617	-37
4	-343,266	-3,801	90
5	286,956	-567	-506
6	-526,747	-2,331	226
7	294,875	-4,697	-63
8	610,737	-4,704	-130
9	-18,561	0	INFINITE
10	-886,553	-3,539	250
11	260,637	-950	-274
12	-817,158	0	INFINITE
13	1,728,919	-8,007	-216
14	432,472	-4,625	-94
15	-221,003	-4,804	46
16	-788,455	-2,015	391
17	-842,611	-1,098	767
18	1,041,500	-5,178	-201
19	326,317	-1,133	-288
20	-542,483	0	INFINITE
21	-417,870	-4,611	91
22	1,343,752	-2,736	-491
23	-373,408	-2,023	185
24	-839,949	-1,199	700
25	1,600,339	-2,614	-612
26	-263,194	0	INFINITE
27	307,697	-7,941	-39
28	176,556	-3,708	-48
29	164,586	-3,068	-54
30	-327,399	-2,831	116
31	-649,530	-2,239	290
32	-611,531	-684	894
33	1,046,320	-8,662	-121
Mean	138,988	-3,548	-39
Median	176,556	-3,268	-54

Table 7. Reallocation from observed to GHG minimizing combination

Category	Corn	Natural Gas	Electricity	Dry	Wet
Measure					
Average Change (%)	-3.05	-6.83	-1.35	-33.63	-4.11

Table 8. Shadow Values: GHG minimizing to ROOC maximizing combination

DMU	WTP for change in allocation, $\pi_*^j - \pi_E^j$ (\$)	Change in GHG emissions, $GHG_*^j - GHG_E^j$ (tons)	Shadow Value of GHG (\$/ton)
1	289,372	650	445
2	1,039,125	579	1,794
3	1,960,763	889	2,206
4	1,567,251	695	2,254
5	332,607	688	484
6	1,789,971	411	4,355
7	1,220,660	597	2,044
8	1,787,797	300	5,964
9	21,760	0	INFINITE
10	1,736,654	903	1,923
11	458,592	687	668
12	818,540	0	INFINITE
13	446,554	298	1,500
14	1,164,994	599	1,945
15	1,972,092	465	4,240
16	1,614,087	988	1,633
17	844,302	1,098	769
18	498,754	622	801
19	904,634	645	1,403
20	800,801	321	2,493
21	2,215,729	886	2,501
22	631,958	701	901
23	1,155,002	1,679	688
24	1,881,661	868	2,168
25	592,059	623	950
26	272,807	0	INFINITE
27	1,993,513	446	4,474
28	1,075,882	632	1,701
29	1,275,255	777	1,641
30	1,433,661	1,030	1,392
31	1,377,339	872	1,580
32	2,008,466	955	2,104
33	818,987	0	INFINITE
Mean	1,243,777	721	1,726
Median	1,220,660	687	1,778

Table 9. Reallocation from GHG minimizing to ROOC-maximizing point

Category \ Measure	Corn	Natural Gas	Electricity	Dry	Wet
Average Change (%)	-2.75	2.82	0.94	12.45	-97.65

Appendix A

The measure in (6) can be mathematically implemented through the following nonlinear programming problem:

$$(A.1) \quad \begin{aligned} & \underset{\lambda, z}{\text{Min}} \lambda \\ & \text{s.t.} \quad \lambda^{-1} u_b^j \leq M_b z, \quad \overline{u_{Eth}^j} = z M_{Eth}, \quad \lambda x^j \geq N z, \quad \sum_j z^j = 1 \end{aligned}$$

Where u_b^j is the vector of dried and wet byproducts, M_b is the $2 \times J$ matrix of observed levels of byproducts, z is the $J \times 1$ vector of intensity variables used to weight observations and construct the piecewise linear boundary of the graph, x^j is the column vector composed by observed values of all inputs used by observation j , N is the $7 \times J$ matrix of observed values of inputs for all observations, and $\overline{u_{Eth}^j}$ is the observed level of ethanol production of the j th DMU.

After multiplying the constraints times λ it is easily seen that this is equivalent to the following problem:

$$(A.2) \quad \begin{aligned} & \underset{\Gamma, z'}{\text{Min}} \Gamma \\ & \text{s.t.} \quad u_b^j \leq M_b z', \quad \Gamma x^j \geq N z', \quad \sum_j z'^j = \lambda, \quad \lambda \overline{u_{Eth}^j} = M_{Eth} z', \quad \Gamma = \lambda^2, \quad z' = \lambda z \end{aligned}$$

Following Färe et al. problem (1) is reformulated into problem (2) because the only nonlinear constraint is an equality constraint (i.e. $\Gamma = \lambda^2$) and is, hence, easier to program. In particular, these sub vector hyperbolic measures of technical efficiency are calculated through a nonlinear program implemented with the FMINCON procedure in MATLAB.

Appendix B

The following program describes the problem:

$$\begin{aligned}
 (B.1) \quad & \text{Min}_{x, u_{DDGS}, u_{MWDGS}} \quad GHG = 0.00668274 x_c + 0.063015823 x_{NG} + 0.0007445 x_{elect} \\
 & \quad - 0.4197522186 u_{DDGS} - 0.407868 u_{MWDGS} \\
 & \text{s.t.} \quad u_{DDGS} \leq M_{DDGS} z, \quad u_{MWDGS} \leq M_{MWDGS} z, \quad \overline{u_{Eth}^j} = M_{Eth} z, \quad x \geq Nz, \quad \sum_j z^j = 1
 \end{aligned}$$

Where u_{DDGS} is the vector of dried byproducts, M_{DDGS} is the 2xJ matrix of observed levels of DDGS, z is Jx1 vector of intensity variables, u_{MWDGS} is the vector of modified wet byproducts, M_{MWDGS} is the 2xJ matrix of observed levels of MWDGS, x is the vector of all inputs, and N is the 7xJ matrix of observed levels of inputs. This program was calculated using the LINPROG routine in MATLAB.

Based on this quantity, we calculate overall environmental efficiency by solving for E_g^j implicitly through Eq. (8) for each observation.

Appendix C

Proof:

Let us denote the vector of coefficients of Eq. (1) by (α_x, β) , where α_x is the vector of coefficients for corn, natural gas, and electricity, and β is the vector of coefficients for both byproducts. In addition, let us define an arbitrary output and input vector by (x_p, u_b) where $x_p = (x_c, x_{NG}, x_{elect})$ and $u_b = (u_{MWDGS}, u_{DDGS})$ and denote the j th DMU's observed output and input vector by (x_p^j, u_b^j) .

Let $(x_p, u_b) \in GHG_g^j \left(E_g^j x_p^j, u_b^j (E_g^j)^{-1} \right) \cap GR$, then $(x_p, u_b) \in GR$ and since E_g^j is a minimum:

$$\begin{aligned} (\alpha_x x_p + \beta u_b) &= E_g^j (0.00668274) x_c^j + E_g^j (0.063015823) x_{NG}^j + E_g^j (0.0007445) x_{elect}^j \\ &\quad - (0.407868) u_{MWDGS}^j / E_g^j - (0.4197522186) u_{DDGS}^j / E_g^j \end{aligned}$$

Let us denote observations j 's minimum feasible GHG level by \underline{GHG}^j . There are three cases to consider:

1. Assume $(\alpha_x x_p + \beta u_b) < \underline{GHG}^j$, then $(x_p, u_b) \notin GR$

2. Assume $\{(\alpha_x x_p + \beta u_b) > \underline{GHG}^j\}$, then

$$\{(v, w) : (\alpha_x v + \beta w) \leq \underline{GHG}^j\} \subseteq \{(v, w) : (\alpha_x v + \beta w) \leq (\alpha_x x_p + \beta u_b)\}$$

and since the hyperplanes defining the two sets are parallel, E_g^j can not be a minimum.

Cases 1 and 2 leave the following case:

3. $(\alpha_x x_p + \beta u_b) = \underline{GHG}^j$. Therefore $(E_g^j \alpha_x x_p^j + E_g^{j-1} \beta u_b^j) = \underline{GHG}^j$.

Appendix D

For a particular DMU i the program is as follows:

Profit Max - fixed observed pn

setting objective function

fi = [confidential data];

linear inequality constraints

Ai=[confidential data];

bi=[confidential data];

linear equality constraints

```
Aeqi=[confidential data];
beqi=[confidential data];
```

optimization procedure

```
options=optimset('LargeScale','off','MaxIter',150);
[xi,fvali] = linprog(fi,Ai,bi,Aeqi,beqi,[],[],[],options);
fvali
xi
```

Appendix E

HISTOGRAM OF SHADOW VALUES

```
A=[-202
-71
-37
90
-506
226
-63
-130
250
-274
-216
-94
46
391
767
-201
-288
91
-491
185
700
-612
-39
-48
-54
116
```

```
290
894
-121];
```

```
histfit(A,20)
```

```
*****
*
```

```
B=[445
1794
2206
2254
484
4355
2044
5964
1923
668
1500
1945
4240
1633
769
801
1403
2493
2501
901
688
2168
950
4474
1701
1641
1392
1580
2104];
```

```
histfit(B,20)
```

CHAPTER 3

ALLOCATION OF ALLOWANCES IN CAP AND TRADE AND PARETO EFFICIENCY

1. Introduction

This study evaluates the validity of assuming Coase's theorem (Coase (1960), Stigler (1966)) in designing allocation of allowances in cap and trade systems. It derives a general relationship between optimal aggregate pollution and allocation of allowances and discusses important ramifications of this link. Of particular interest is the result that shows that allocating allowances (or revenues from their selling or auctioning) to low income households seems to enhance efficiency.

There is widespread scientific evidence that natural ecosystems might not adapt to the rapid changes in climate caused by global warming. These changes are expected to include compromised survivability of plants and animals, altered weather patterns, increased sea levels, spread of human diseases (especially tropical diseases) and decreased crop yields. Most emissions of greenhouse gases (GHG) come from the burning of fossil fuels for energy and the clearing of land by individuals and firms. Therefore, atmospheric concentration of CO₂ can be seen as a *privately produced* public bad.

Price and quantity controls can be used to internalize the social costs of this public bad. The existence of compliance and cost uncertainty leads to different welfare outcomes for the two policy instruments. Weitzman (1974) derived theoretical conditions under which one policy is preferred to the other. For a variety of reasons, political feasibility not being

the least of them, the majority of countries have chosen quantity controls as the instrument of choice for dealing with this problem.⁴⁰ We analyze in this study the optimal design of this instrument but we do so under the assumption of certainty. Therefore there are no welfare implications to the choice of quantity control over price control.⁴¹

A quantity control implies assigning property rights on pollution to different agents in the economy. The assignment of property rights allows for creation of a market through which the externality can be traded. If rights are allocated to polluters (situation known as “permissive” law) then pollutees (households) pay polluters (firms) to reduce pollution. On the other hand if the law is prohibitive (pollutees own the rights) the polluters pay to the pollutees for the right to pollute. The distribution of permits across and within groups is usually deemed irrelevant on efficiency grounds. Coase’s theorem is invoked to support such a conclusion; i.e. regardless of how permits are distributed, agents will achieve the efficient pollution level through decentralized trading. However under current designs of cap and trade systems the overall level of pollution is determined exogenously by law rather than through trading⁴² and hence it is not clear whether Coase’s theorem can be invoked.

Although a generally accepted definition of Coase’s theorem remains unsettled there are two principles that characterize it: efficiency and invariance. The former states that creation of property rights will result in an efficient outcome regardless of whom those

⁴⁰ Setting standards on technology or permissible emissions is another potential instrument that governments could use to tackle this problem.

⁴¹ This instrument was not only chosen by countries to tackle national policy but also by groups of countries to create international environmental agreements such as the Kyoto protocol.

⁴² For example the HR.2454: American Clean Energy and Security Act of 2009 establishes a cap of 17% below 2005 by 2020 and 80% below 2005 by 2050.

<http://www.epa.gov/climatechange/economics/pdfs/WaxmanMarkeyExecutiveSummary.pdf>

rights are assigned to.⁴³ The latter principle states that different allocations of property rights will not only result in an efficient outcome but the same efficient outcome (i.e. same quantity and prices).

In the context of cap and trade systems, alternative allocations of allowances are discussed in Congress in relation to their impact on income distribution. Some bills (e.g. HR.2454: American Clean Energy and Security Act of 2009 also known as Waxman-Markey or WM for short) allocate an increased portion of allowances⁴⁴ to low income households⁴⁵ to enhance equity in income distribution while others do not. Assessment of alternative allocations of allowances has been conducted only on normative grounds. They are assumed to affect income distribution but not the optimal level of the cap (and hence efficiency). The rationale usually put forth for this is the invariance principle involved in Coase's theorem. If the invariance principle does not hold then changes in income distribution (presumably through reallocation of allowances) while keeping the cap fixed may cause the economy to deviate from the Pareto efficient solution and incur welfare losses. This point is illustrated in Figure 1.

Figure 1 displays marginal social benefits and costs of pollution. Benefits from pollution come from production of final goods that are consumed by households. Social costs come from negative effects of climate change. Suppose the cap is fixed at a level which corresponds to the Pareto efficient level, P^0 . An increase in the portion of

⁴³ As long as transaction costs are low enough.

⁴⁴ In cap and trade systems allowances can be directly allocated to certain groups of households or they can be sold or auctioned by the government and the proceeds transferred to households through lump sum transfers. We will use the expressions "allocating allowances" and "transferring revenues from allowance" interchangeably.

⁴⁵ WM allocates 45% of the value of allowances (approximately \$30 billion) to consumers emphasizing low income consumers while a recently announced proposal by Senators Kerry and Lieberman maintains the same cap as WM (see footnote 43) but it increases transfers of revenue (from permits selling) to low income households through consumer rebates. The proposal establishes that a 75% of revenues from auctioning of permits will be allocated to consumers in 2026.

allowances allocated to low income households is considered by the Congress to enhance equity in income distribution maintaining the cap at P^0 .

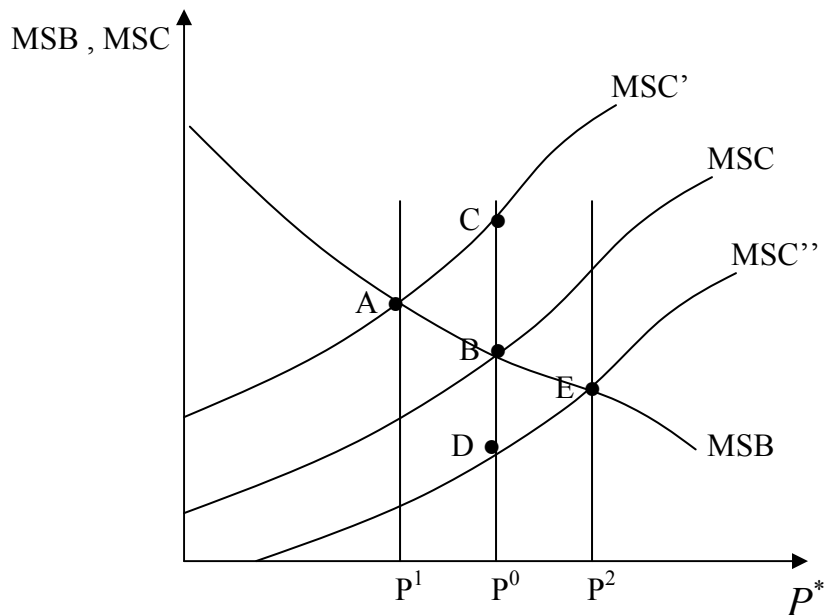


Figure 1. Allowance allocation, efficiency, and welfare

Starting from the optimal pollution level if an increase in the portion of allowances allocated to low income households changes the optimal level of pollution from P^0 to P^1 (e.g. because low income consumers significantly increase their preference for a cleaner atmosphere when their incomes are increased) then a welfare loss is incurred. The welfare loss amounts to the area ABC. If, on the other hand, an increase in allowances allocated to low income households increases the level of optimal pollution from P^0 to P^2 then the welfare loss is equivalent to the area DBE.

As illustrated by Figure 1 there is a link between the optimal level of pollution and income distribution. This link may in turn create a relationship between efficiency (i.e. existence of welfare losses) and equity. This study will investigate the link between allocation of allowances to low income households and optimal pollution underlying

Figure 1. We start by discussing the potential influence of allocation of allowances on income and, hence, the social cost of pollution. This is followed by a description of the economy under study. We then depict the social optimum level of pollution in this economy. Following this we describe how a cap and trade system would work in this economy and what the solution of a decentralized economy looks like. The latter will allow us to derive values of policy instruments that would achieve the Pareto efficient allocation in a market economy. Finally the link between efficiency and income distribution is inferred from the relationship between policy instruments.

2. Allocation of Allowances, Income Distribution, and Optimal Cap

The only case in which distribution of allowances can be discussed without concerns about efficiency and welfare is when the invariance principle of Coase's theorem holds. This is possible whenever the effect of changes in allocation of allowances on income distribution is zero or when changes in income distribution do not affect optimal aggregate pollution. However, we need to consider if these facts are plausible or not.

The WM bill proposes the creation of 4.6 billion allowances in the first year. According to EPA's preliminary estimates of the implications of WM the price of a permit to emit one ton of carbon dioxide or its equivalent would range between \$13 and \$17 (in 2005 dollars) in the first stage after the bill takes effect. Assuming a price of \$15 which is at the average of the estimated range, the total value of allowances in the first year would amount to about 69 billion dollars. Based on these values Table 1 shows how allocating permits to different income categories of consumers in the US may affect their income, their demand for energy (we use energy as a proxy for a broader concept of

consumption), and hence the associated optimal aggregate level of pollution. For simplicity Table 1 assumes constant income and price elasticity of demand. This is not necessarily supported by empirical evidence but we use it here for the purpose of illustration.

Table 1. Income Effects and Welfare Losses under Different Permit Distributions

	GDP (billions 2008)	Highest Quintile (billions 2008)	Four Lowest Quintiles (billions 2008)	Three Lowest Quintiles (billions 2008)	Two Lowest Quintiles (billions 2008)	Lowest Quintile (billions 2008)
Income ¹	14,200	6,958	7,242	3,976	1,846	511.2
Income Change (%) ²	0.49	1	0.96	1.75	3.76	13.58
Income Elasticity of Demand for Energy ³	1	1	1	1	1	1
Energy Price Change (%) ⁴	7	7	7	7	7	7
Price Elasticity of Demand for Energy ⁵	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
Change in Energy Demand (%) ⁶	-2.31	-1.8	-1.84	-1.05	0.96	10.78

¹ Income, Poverty, and Health Insurance Coverage in the United States: 2008

<http://www.census.gov/prod/2009pubs/p60-236.pdf>

² These values are calculated as the percentage increase in income associated with allocating the value of all allowances to each group; $\Delta \text{income}^i / \text{income}^i$ where Δincome^i is equal to the total value of allowances and income^i is the level of income of household group i .

³ This assumption is consistent with that of most general equilibrium models of the US economy; e.g. EPPA, CETA, DICE-99, MERGE, SGM (Webster et al.)

⁴ EIA estimation: Waxman- Markey (baseline case)

⁵ This assumption is consistent with that of general equilibrium models of the US economy; e.g. EPPA, ER, and MERGE (Webster et al.)

⁶ This assumption is consistent with that of most general equilibrium models of the US economy; e.g. EPPA, CETA, DICE-99, MERGE, SGM (Webster et al.)

Despite the (potential) qualitative and quantitative relevance of allowance allocation in aggregate pollution and total welfare no studies exploring the link between optimal level of emissions and permits distribution can be found in the literature and this paper attempts to fill that gap. The present study investigates the link between income distribution and optimal aggregate pollution and analyzes the implications of such a link for the simultaneous achievement of economic efficiency (choosing the allowance

distribution that rationalizes the exogenous cap) and equity (choosing among allowance distributions that reduce the income gap across income categories). In particular necessary and sufficient conditions for complementarity, independence and substitutability between income distribution and efficiency are derived and expressed in terms of technological, preference and ecological parameters.

3. The Economy

We will investigate now the link between allocation of allowances to low income households and optimal pollution underlying Figure 1. The optimal choice of pollution is essentially a dynamic problem. In any given period economies choose to emit carbon to the atmosphere to produce a consumption good which in turn increases consumers' utility. However, in the following periods, emissions accumulate in the atmosphere and the stock of pollutants causes disutility to consumers. Therefore modeling optimal pollution requires a dynamic model of the economy. The main goal of this study is to explore the (dynamic) relationship between income distribution and optimal pollution and its implications in terms of the competing (trade off) or complementary nature of efficiency and equity goals. Therefore our model of the economy is one of households with different levels of income and pollution generating production.

3.1. Production

The economy produces a good that can be consumed or saved as capital. Production requires the use of energy which in turn results in the emission of greenhouse gases to the atmosphere. We follow Copeland and Taylor (1994), (1999), and Stokey (1998) and

model pollution as an input to production.⁴⁶ The technology for production of this good is represented by:

$$y_t = f_t(k_t, e_t(p_t)) \quad (1)$$

Where y_t is output at time t , k_t is capital used at t and p_t is emissions at t which is a function of energy used at t , e_t . We will denote the function relating energy to pollution by $p_t(e_t)$. This function depicts the level of emissions associated with using e_t of energy. The inverse of this function denotes the level of energy associated with a given flow of emissions and we denote it by $e_t(p_t)$. The production function depicted by (1) is assumed to be twice continuously differentiable in both arguments. This technology allows for substitution between capital and energy but since it considers one aggregate index of energy it does not consider substitution between different sources of energy.

The capital stock k_t evolves according to the following equation of motion:

$$k_{t+1} - k_t = f_t(k_t, e_t(p_t)) - x_t^i - \mu k_t \quad (2)$$

Where μ is the depreciation rate.

Emissions accumulate through time to form a pollution stock P_t (e.g. concentration of GHGs in the atmosphere expressed as parts per million):

$$P_{t+1} = p_t + P_t(1 - \lambda) \quad (3)$$

Where λ is the natural rate of dissipation and p_t are emissions at t .

⁴⁶ This is equivalent to modeling pollution as a byproduct with a technology in which pollution and other inputs are separable from the final product.

3.2. Consumption

Consumers are assumed to be homogeneous in terms of their preferences but heterogeneous with respect to their income and initial wealth. To simplify the analysis we limit consumers' sources of income to interests on initial holdings of capital and, later, on income from sales of emission permits. We ignore labor supply in this model which, although counterintuitive, does not diminish the insights of the model.

There are N households in the economy and no population growth. Consumers' intertemporal preferences are characterized by a discounted utility function of the form:

$$U^i(x^i, P) = \sum_{t=0}^{\infty} \delta^t u_i(x_{it}, P_t). \text{ Since we are assuming homogeneity in preferences we can}$$

drop the subscript i on utility and re express it as:

$$U(x^i, P) = \sum_{t=0}^{\infty} \delta^t u(x_{it}, P_t) \quad (4)$$

Where δ is the subjective discount rate, x_{it} is the consumption level of individual i at time t , and P_t is the accumulated stock of pollution at time t

The per-period utility function $u(x_{it}, P_t)$ is assumed twice continuously differentiable in both arguments.

In this model emissions are not directly caused by consumption but the stock of pollution causes disutility to the consumer. Although household behavior is undoubtedly an important source of pollution (e.g. driving cars), empirically, the majority of GHG emissions come from the production sector of the economy. So to simplify matters we ignore household-generated pollution and just focus on carbon emitted during the production of capital and final consumption goods.

4. Social Optimum

We assume that the social planner chooses a level of the externality along with the “private” variables to maximize a given social welfare function. As it is conventional in this framework, let $\Delta = \left\{ \eta_1, \dots, \eta_n \mid \eta_i \geq 0 \text{ and } \sum_{i=1}^n \eta_i = 1 \right\}$ be the unit simplex of \Re^n . A Pareto Optimal Allocation in this economy is a solution to the planner’s problem for a given vector of welfare weights $\eta = (\eta_1, \dots, \eta_n) \in \Delta$. The planner’s problem is:

$$\max_{\{x_{it}, y_t, P_t\}_{t \geq 0}} \sum_{i=1}^n \eta_i \sum_{t=0}^{\infty} \delta^t u(x_{it}, P_t)$$

$$s.t. \quad (2) - (3)$$

$$\sum_{i=1}^n x_{it} = x_t = f_t(k_t, p_t(e_t)) \quad (5)$$

$$k(0) = k_0 \quad (6)$$

Constraint (5) is a “no resource waste” condition. The entire production is consumed by households and hence no production is wasted.

We can now find an instantaneous return function (or value function) for the social planner. This function is just the maximized weighted sum of individual returns:

$$u(x_t, P_t) = \max_{\{x_{it}, P_t\}_{t \geq 0}} \sum_{i=1}^n \eta_i u(x_{it}, P_t) \quad (7)$$

$$s.t. \quad \sum_{i=1}^n x_{it} = x_t$$

Considering that $x_t = f_t(k_t, e_t(p_t))$ and plugging (3) in the second argument of the production function respectively yields:

$$x_t = f(k_t, e_t(P_{t+1} - P_t(1 - \lambda)))$$

We now define the indirect utility function in terms of the production function as:

$$V(k_t, k_{t+1}, P_t, P_{t+1}) = u \left[f(k_t, e_t(P_{t+1} - P_t(1-\lambda))) , P_t \right] \quad (8)$$

Using the two equations of motion (2) and (3) it can be easily shown that the set of admissible paths form a compact and convex set \mathfrak{T} (see Appendix for proof). This means that that indirect utility function is a continuous function defined on a compact and convex domain which in turn allows us to express the planner's problem in terms of this function $V(k_t, k_{t+1}, P_t, P_{t+1})$ as:

$$\max_{\{k_t, P_t\}_{t \geq 0}} \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t+1}, P_t, P_{t+1}) \quad (9)$$

$$s.t. \quad (k_t, k_{t+1}, P_t, P_{t+1}) \in \mathfrak{T} \text{ and } k(0) = k_0$$

The solution of the planner's problem is characterized by the sequence $\{k_t, P_t\}_{t \geq 0}$ that constitutes a solution to the following system of Euler equations (subscripts refer to the associated partial derivatives):

$$\begin{aligned} V_2(k_t, k_{t+1}, P_t, P_{t+1}) + \delta V_1(k_{t+1}, k_{t+2}, P_{t+1}, P_{t+2}) &= 0 \\ V_4(k_t, k_{t+1}, P_t, P_{t+1}) + \delta V_3(k_{t+1}, k_{t+2}, P_{t+1}, P_{t+2}) &= 0 \end{aligned} \quad (10)$$

And that satisfy the transversality conditions

$$\lim_{t \rightarrow \infty} \delta k_t V_1(k_t, k_{t+1}, P_t, P_{t+1}) = 0$$

$$\lim_{t \rightarrow \infty} \delta P_t V_3(k_t, k_{t+1}, P_t, P_{t+1}) = 0$$

Denoting the derivative of the indirect utility function with respect to i at period t u_i^t

and using the function in equation (8) and plugging in (10) yields:

$$\begin{aligned}
& -u'_{x_t}(x_t, P_t) + \delta u'_{x_t}(x_{t+1}, P_{t+1}) \left[f_{k_{t+1}}(k_{t+1}, p_{t+1}(e_{t+1})) + (1 - \mu) \right] = 0 \\
& u'_{x_t}(x_t, P_t) f_{e_t}(k_t, e_t(p_t)) e_{p_t}(p_t) - \delta u'_{x_t}(x_{t+1}, P_{t+1}) f_{e_{t+1}}(k_{t+1}, p_{t+1}(e_{t+1})) e_{p_{t+1}}(p_{t+1}) (1 - \lambda) + \\
& \frac{u'_P(x_t, P_t)}{(1 - \lambda)} + \delta u'_P(x_{t+1}, P_{t+1}) = 0
\end{aligned} \tag{11}$$

System (11) denotes the necessary conditions for a socially optimal allocation in terms of the two stock variables k_{t+1} and P_{t+1} . The first equation states that the utility derived from an additional unit of current consumption must equal the utility obtained by investing that unit and consuming the proceedings of that investment in the next period. The second equation is the well known Lindahl-Bowen-Samuelson condition for efficiency under externalities. This condition states that the marginal social benefits of pollution coming from the use of energy for production, must equal its marginal social cost resulting from namely effects of GHG concentration on health, biodiversity, weather extremes, and natural disasters.

We are mainly concerned with the socially optimum allocation at the steady state and the effect that distribution might have on steady state level of pollution. At steady state $x_t = x_{t+1} = x^*$, $k_t = k_{t+1} = k^*$ and $P_t = P_{t+1} = P^*$. From the equations of motion at steady state we know that $x_i^* = f(k^*, e^*(p^*)) - \mu k^*$ $i=1,2$ and $p^* = \lambda P^*$. Necessary conditions for a social optimum at steady state can be expressed as:

$$\begin{aligned}
& \delta \left[f_k(k^*, e^*(p^*)) + (1 - \mu) \right] = 1 \\
& u'_{x^*}(x^*, P^*) f_{e^*}(k^*, e^*(p^*)) e_{p^*}(p^*) \\
& - \delta u'_{x^*}(x^*, P^*) \left[f_{e^*}(k^*, e^*(p^*)) e_{p^*}(p^*) \right] (1 - \lambda) + \frac{u'_P(x^*, P^*)}{(1 - \lambda)} + \delta u'_P(x^*, P^*) = 0
\end{aligned} \tag{12}$$

We can now link the optimal aggregate level with the individual levels of consumption through the expressions for $u_x(x^*, P^*)$ and $u_P(x^*, P^*)$. The indirect utility function at steady state is:

$$u(x^*, P^*) = \sum_{i=1}^N \eta_i u(x_i^*, P^*) = \sum_{i=1}^{N-1} \eta_i u(x_i^*, P^*) + \eta_N u\left(x^* - \sum_{i=1}^{N-1} x_i^*, P^*\right)$$

Where $\eta_N u\left(x^* - \sum_{i=1}^{N-1} x_i^*, P^*\right)$ represents the weighted utility of the N th household.

The derivatives are:

$$u_{x_i}(x^*, P^*) : \eta_i u_{x_i}(x_i^*, P^*) - \eta_N u_{x_N}\left(x^* - \sum_{i=1}^{N-1} x_i^*, P^*\right) = 0 \quad \forall i \neq N \quad (13.a)$$

$$u_x(x^*, P^*) = \eta_N u_x\left(x^* - \sum_{i=1}^{N-1} x_i^*, P^*\right) \quad (13.b)$$

Combining (13.a) and (13.b) yields:

$$u_x(x^*, P^*) = \eta_i u_{x_i}(x_i^*, P^*) \quad \forall i \quad (13.c)$$

These expressions indicate that, at the optimum, the social planner equalizes weighted marginal utilities of all consumers.

In addition the derivative of $u(x^*, P^*)$ with respect to the overall level of pollution is:

$$u_P(x^*, P^*) = \sum_{i=1}^N \eta_i u_P^i(x_i^*, P^*) \quad (13.d)$$

The relationship in (13c) reveals that the system (12) really involves $N+1$ equations:

$$\begin{aligned}
& \left[f_k(k^*, e^*(p^*)) + (1-\mu) \right] - \frac{1-\delta(1-\mu)}{\delta} = 0 \\
& - \frac{\eta_i u_{x_i}(x_i^*, \lambda^{-1} p^*)}{\sum_{i=1}^N \eta_i u_p^i(x_i^*, \lambda^{-1} p^*)} - \frac{1}{(1-\lambda)} \frac{\left(\frac{1+\delta(1-\lambda)}{1-\delta(1-\lambda)} \right)}{\left[f_e(k^*, e^*(p^*)) e_p(p^*) \right]} = 0 \quad \forall i=1, \dots, N
\end{aligned} \tag{14}$$

This system is defined in terms of p^* , k^* , and x_i^* , $i=1, \dots, N$. There are however only N independent variables as the N th x_i^* is determined by the no-waste condition

$$f(k^*, e^*(p^*)) = \sum_{i=1}^N x_i^*.$$

5. Market Economy

Levels of production, capital, energy and pollution result from the behavior of consumers and firms which are in turn driven by prices in markets. We turn our attention now to modeling this behavior of consumers and producers.

5.1. Producers Problem under Cap and Trade

Now we concern ourselves with allocative efficiency in the production sector of the economy. In particular we express the firms' profit maximization problem in terms of the production possibilities frontier as:

$$\text{Max } \Pi = f(k_t, e_t(p^0)) - rk_t - p_p p^0 - p_e e_t(p^0)$$

Where: $f(k_t, e_t(p^0))$ is the level of production at t

r is the rental rate of capital

p_p is the price of a permit

p_e is the price of energy

p^0 is the cap on emissions established by the government

Note that profits Π are written in the same way regardless of whether the firm is endowed with the permits or whether it has to buy them in the market. In both cases the term $(-p_p p^0)$ is included in Π but its interpretation differs. If firms have to buy permits in the market this term captures actual payments in pollution permits. If permits are allocated to the firms this term captures the opportunity cost of using the permits in production rather than selling them in the market at price p_p . So the manner in which permits are allocated to firms makes no difference in terms of their decisions and market outcome.

There are no private intertemporal trade-offs faced by firms and hence we can drop the time subscripts. From the first order conditions of this problem (levels of variables resulting from private optimization will be denoted by the superscript M) we derive the following relationships:

$$f_k(k_t, e_t(p^0)) = r \quad (15)$$

$$f_e(k_t, e_t(p^0)) \frac{de_t}{dp^0} = p_p + p_e \frac{de_t}{dp^0} \quad (16)$$

The combination of equations (19) and (20) determines the derived demands for capital and pollution/energy as functions of prices.

5.2. Consumers Problem under Cap and Trade

In our model consumers maximize the present value of their future utility stream subject to their budget constraint. In a market economy consumers use their income for

consumption or savings. The level of income of each household i at time t is given by an exogenous source denoted by I_{it} and transfers from the government ε_{it} funded by revenues from permits selling. Consumers save by accumulating assets and the accumulated level of this asset at time t is denoted by A_{it} . Meanwhile the level of consumption is denoted by x_{it} . Therefore the consumer's problem is as follows:

$$\begin{aligned} \underset{x_{it}, P_{it}}{\text{Max}} \quad & U(x^i, P) = \sum_{t=0}^{\infty} \delta^t u(x_{it}, P_t) \\ \text{s.t.} \quad & A_{it+1} = (1 + r_t) A_{it} + I_{it} + \varepsilon_{it} - x_{it} \end{aligned} \quad (17)$$

Where r_t is the market interest rate, A_{it} denotes assets held by household i at time t , I_{it} is income of household i at t , ε_{it} is additional income transferred from the government to household i at time t (funded by the proceeds from the selling or auctioning of permits to producers),⁴⁷ and x_{it} is consumption of household i at t .

Solving problem (17) yields the following first order conditions:

$$\begin{aligned} u_{x_{it}}(x_{it}^M, P_t) &= \delta(1 + r_t) u_{x_{it+1}}(x_{it+1}^M, P_{t+1}) \\ A_{it+1} &= (1 + r_t) A_{it} + I_{it} + \varepsilon_{it} - x_{it} \end{aligned}$$

Solving for optimal consumption and savings (x_{it} and A_{it+1}) requires combining both FOCs and solving the resulting difference equation. In steady state the solution can be generically denoted by:

$$x_i = c_i(r, I_i, \varepsilon_i) \quad (18)$$

Therefore steady state consumption is a function of the interest rate, the household's level of income, and income transfers from the government to the household. The

⁴⁷ Instead of directly allocating permits to households we assume that the government sells or auctions the permits to producers (covered sectors). The proceeds from this are then distributed across households through lump sum transfers. This is consistent with most mechanisms discussed in climate bills.

function $c_i(r, I_i, \varepsilon_i)$ can be linear or non-linear in income and transfers and if it is non-linear it can be concave or convex depending upon preferences.

5.3. Market Equilibrium Conditions

Behavior of firms and consumers are functions of prices. Prices on the other hand are determined endogenously in a market economy based on market equilibrium conditions and the identity that establishes that revenue from permits selling or auctioning is rebated through lump sum transfers to consumers (budget equilibrium condition):

$$k(r, p_p, p_e, p^0) = \sum_{i=1}^N k_i(r, I_i, \varepsilon_i) \quad (19.a.)$$

$$f(k(r, p_p, p_e, p^0), e(p^0)) = \sum_{i=1}^N c_i(r, I_i, \varepsilon_i) \quad (19.b.)$$

$$p^0 p_p = \sum_{i=1}^N \varepsilon_i \quad (19.c.)$$

Based on these equations we will find expressions for equilibrium prices as functions of exogenous variables namely income levels $I = \{I_i\}_{i=1}^N$, and also as functions of policy instruments $\varepsilon = \{\varepsilon_i\}_{i=1}^N$ and p^0 :

$$r(p^0, \varepsilon, I) \quad (20)$$

$$p_e(p^0, \varepsilon, I) \quad (21)$$

$$p_p(p^0, \varepsilon, I) \quad (22)$$

These functions capture in some sense the private reaction to public policy and their impact on market equilibrium. Therefore equilibrium prices can be used to discuss the implementation of the social optimum in (17) through a cap and trade system.

6. Income Distribution and Optimal Pollution

6.1. Implementation of Social Optimum in a Market Economy

Necessary conditions for a social optimum were depicted in (14). For simplicity we will assume two groups of households, groups 1 and 2. Under this assumption (and dropping the * superscript) system (14) becomes:

$$f_K \left[K, E(p^0) \right] = \frac{1 - \delta(1 - \mu)}{\delta}$$

$$- \frac{\eta_1 u_{x_1}(x_1^*, \lambda^{-1} p^0)}{\sum_{i=1}^N \eta_i u_p^i(x_i^*, \lambda^{-1} p^0)} - \left(\frac{1 + \delta(1 - \lambda)}{1 - \delta(1 - \lambda)} \right) \frac{1}{(1 - \lambda)} \frac{1}{f_E \left[K, E(p^0) \right] E_p} = 0$$

$$\eta_1 u_{x_1}(x_1^*, \lambda^{-1} p^0) = \eta_2 u_{x_2}(x_2^*, \lambda^{-1} p^0)$$

Which after plugging the first and third equations into the second yields:

$$- \frac{\eta_2 u_{x_2}(x_2^*, \lambda^{-1} p^0)}{\sum_{i=1}^N \eta_i u_p^i(x_i^*, \lambda^{-1} p^0)} - \left(\frac{1 + \delta(1 - \lambda)}{\delta(1 - \lambda)} \right) \frac{1}{f_K \left[K, E(p^0) \right] f_E \left[K, E(p^0) \right] E_p} = 0 \quad (23)$$

In this context the social planner determines aggregate capital and pollution, and individual households' consumption. In reality, however, the government creates a market for emissions and only controls the overall level of pollution and lump sum transfers to individual households. So in a decentralized economy the government, anticipating the reaction of private agents through markets, aims at implementing conditions described in (23) through manipulation of the aforementioned policy instruments. This can in fact be thought as a dynamic game between the government and private agents in which the government moves first and hence the game is solved through backward induction.

The solution to the government's problem of maximizing social welfare in a decentralized market economy consists of expressions that depict levels of policy instruments as functions of exogenous variables namely initial wealth distribution. To find such a solution we need to incorporate firm's behavior (15) and (16), consumers' behavior (18), and equilibrium prices (20)-(22):

$$\overbrace{\frac{\left(\frac{1+\delta(1-\lambda)}{\delta(1-\lambda)}\right) \sum_{i=1}^2 \eta_i u_p^i \left(c_i \left(\varepsilon_i, I_i, r(p^0, \varepsilon, I)\right), \lambda^{-1} p^0\right)}{r(p^0, \varepsilon, I)}}^{\text{MSC of pollution}} = \overbrace{\left[p_p(p^0, \varepsilon, I) + p_e(p^0, \varepsilon, I) \frac{de_t}{dp^0} \right] \eta_2 u_{x2} \left(c_2 \left(\varepsilon_2, I_2, r(p^0, \varepsilon, I)\right), \lambda^{-1} p^0\right)}^{\text{MSB of pollution}} \quad (24)$$

As discussed in the introduction to this study the link between equity and efficiency in the context of a cap and trade system critically depends upon the relationship between income distribution and optimal pollution. If there is no link between income distribution and optimal pollution Coase's theorem will hold. In this case the allocation of allowances can be used as an instrument for income redistribution while keeping the cap fixed without violating Pareto efficiency. If a link between income distribution and optimal pollution exists, on the other hand, Coase's theorem does not hold and any change in allowance allocation while keeping the cap fixed would entail a violation of Pareto efficiency and hence a welfare loss.

The link between income distribution and the optimal level of aggregate steady state pollution can be derived from equation (24). System (24) implicitly defines levels of the cap (p^0) and distribution of revenue from permit selling $(\varepsilon_1, \varepsilon_2)$ that would achieve a Pareto efficient allocation in a market economy. After imposing the government's budget

equilibrium condition ($\varepsilon_2 = p^0 p_p(p^0, \varepsilon, I) - \varepsilon_1$) we denote the implicit function in (24) by $G(p^0, \varepsilon_1) = 0$. This expression shows that, given wealth distribution and private behavior, changes in p^0 would require changes in ε_1 to maintain Pareto efficiency and maximize social welfare. An important corollary of (24) is that there is no unique efficient level of pollution. A rather wide range of levels of pollution can be rationalized by fine tuning the distribution of revenue through lump sum transfers ε_1 .

6.2. Distribution of Allowances and Optimal Pollution

Transfers of revenue from permit selling can be conducted without any regard for efficiency as long as no link exists between these transfers and the optimal level of aggregate pollution. This will hold true whenever the invariance principle of Coase's theorem obtains, i.e. $\frac{d\varepsilon_1}{dp^0} = 0$. On the other hand if this does not hold, revenue transfers will affect the optimal level of pollution creating a wedge between this level and the exogenous cap and resulting in deadweight losses.

If the invariance principle does not hold there could be a positive or a negative relationship between pollution abatement (reduction in p^0) and income distribution. There will be a complementarity (trade off) between abatement and equity whenever higher (lower) transfers to low income households are needed to rationalize a lower level of pollution. If household type 1 earns a low level of income and $\frac{d\varepsilon_1}{dp^0} < 0$ then both

income redistribution and pollution abatement can be achieved simultaneously. We can

say then that there is a complementarity between goals. There is a trade off if $\frac{d\varepsilon_1}{dp^0} > 0$.

We denote the income elasticity of demand corresponding to households 1 and 2 as ω_1 and ω_2 respectively. We derive the expression for $\frac{d\varepsilon_1}{dp^0}$ by applying the implicit function

theorem to $G(p^0, \varepsilon_1)$ in equation (24):

$$\frac{d\varepsilon_1}{dp^0} = -\frac{G_{p^0}(p^0, \varepsilon_1)}{G_{\varepsilon_1}(p^0, \varepsilon_1)} = \frac{\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\} + \left\{ \gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi \right\}}{-\left\{ \gamma \left[\frac{dc_2}{d\varepsilon_1} \right] \right\} + \left\{ \kappa \left[\frac{dc_1}{d\varepsilon_1} \right] + \alpha \left[\frac{dc_2}{d\varepsilon_1} \right] - \psi \right\}} \quad (25)$$

Where:

$$\frac{dc_1}{d\varepsilon_1} = \frac{c_1}{\varepsilon_1} \omega_1 + \frac{dc_1}{dr} \frac{dr}{d\varepsilon_1}, \quad \frac{dc_2}{d\varepsilon_1} = \frac{c_2}{\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) \omega_2 + \frac{dc_2}{dr} \frac{dr}{d\varepsilon_1},$$

$$\alpha = \frac{\eta_2^2 u_{c_2} u_{pc_2}}{u_p^2}, \quad \beta = \frac{\eta_2 u_{c_2} \eta_1 u_{pc_1}}{u_p^2}, \quad \gamma = \frac{\eta_2 u_{c_2 c_2}}{u_p}, \quad \phi = \frac{\eta_2 u_{c_1 p}}{u_p}, \quad \tau = \frac{\eta_2 u_{c_2} u_{pp}}{u_p u_p}$$

$$\chi = \frac{\eta_2 u_{c_2}}{u_p} \left[\frac{\frac{dr}{dp^0} (p_p + e_p p_E) + r \left(\frac{dp_p}{dp^0} + e_{pp} p_E + e_p \frac{dp_E}{dp^0} \right)}{r^2 (p_p + e_p p_E)^2} \right],$$

$$\kappa = \frac{\eta_2 u_{c_2}}{u_p} \frac{\eta_1 u_{pc_1}}{u_p}, \quad \psi = \frac{\eta_2 u_{c_2}}{u_p} \left[\frac{\frac{dr}{d\varepsilon_1} (p_p + e_p p_E) + r \left(\frac{dp_p}{d\varepsilon_1} + e_p \frac{dp_E}{dp^0} \right)}{r^2 (p_p + e_p p_E)^2} \right]$$

Under usual regularity assumptions (concavity of utility and negative cross derivatives

u_{pc_1} and u_{pc_2}) $\alpha < 0$, $\beta < 0$, $\gamma > 0$, $\phi > 0$, $\kappa < 0$, and $\tau < 0$.

Our next step is to disentangle the conditions under which Coase's theorem can be invoked and indentify, when Coase's theorem does not hold, whether environmental and distributional goals are substitutes or complements.

6.2.1. Independence of Goals and Fulfillment of Coase's Theorem

We find in this section conditions for independence between pollution abatement and income distribution which in turn imply fulfillment of Coase's theorem.

PROPOSITION 1. There is independence between permit distribution and optimal pollution whenever:

1. Substitution effects in demand for consumption good and market effects⁴⁸ of changes in the cap are such that the change in marginal social benefit are exactly the same as the

change in marginal social cost; i.e. $\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\} = - \left\{ \gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi \right\}.$

2. Preferences, pollution-energy relationship, and market effects are such that changes in the cap cause no effect on marginal social benefit and marginal social cost. In particular no effect of changes in the cap on interest rates (i.e. $\frac{dr}{dp^0} = 0$) and preferences that are

linear in pollution (i.e. $\tau = 0$) imply no effect of a cap change on marginal social benefit

$\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\}.$ Additionally preferences that are additively separable in

consumption and pollution (i.e. $u_{c_1 p} = 0$ which results in $\gamma = \phi = 0$), and no market effects of changes in the cap and a linear relationship between energy and pollution (i.e.

⁴⁸ We refer to effects of changes in the cap on equilibrium market prices as "market effects."

$\frac{dr}{d\varepsilon_1} = \frac{dp_p}{d\varepsilon_1} = \frac{dp_E}{d\varepsilon_1} = e_{pp} = 0$ which jointly result in $\chi = 0$) imply no effect of a cap change

on marginal social cost $-\left\{\gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi\right\}$.

3. Income elasticity of consumption of group 1 or market effects are so high that the effect of a change in the cap on marginal social cost is infinite. In addition no link between the cap and allocation exists if income elasticity of consumption of group 2 is so high that the effect of a change in the cap will make marginal social benefit (cost) infinite (i.e. $\gamma > (<) \alpha$).

See proof in Appendix.

Condition 1 depicts a situation in which benefits of reducing pollution (reduction of disutility from pollution for households 1 and 2) are exactly outweighed by welfare loss of consumption reduction, resulting from changes in prices and income.

Condition 2 is self explanatory. If changes in the level of pollution cause no change in prices and, in addition, changes in income have no effect on consumption, the way revenues are distributed is irrelevant. Whether permits are allocated to group 1 or 2 will not matter for aggregate market equilibrium and consumers' welfare.

Finally if consumption tends to be perfectly elastic with respect to income for all households then revenue distribution is irrelevant. This is due to the fact that transfers of permits to either group will have the same effect on consumption and welfare.

Are these conditions plausible? Condition 1 is implausible since it requires specific values of price elasticity of consumption for both groups of households and of income elasticity of consumption of type 2 households. Condition 2 requires perfectly inelastic demands with respect to both prices and income and no effect on market prices of

changes in the cap. This is also very unlikely. Finally condition 3 requires consumption functions that are infinitely elastic with respect to income. This is hardly the case if we look at empirical estimates of Engel curves in the United States.⁴⁹

To sum up conditions required for orthogonality between permit distribution and optimal pollution are unlikely to hold. This in turn implies that the invariance principle embedded in Coase's theorem is inappropriately invoked when discussing allocation of allowances in cap and trade design and also when conducting comparisons between cap and trade and carbon taxes.

6.2.2. Trade off or Complementarity between Goals

There will be a complementarity (trade off) between pollution abatement and equity whenever $\frac{d\varepsilon_1}{dp^0} < (>) 0$ and household group 1 earns a low level of income. In order to identify whether group 1 is a high or low income group we will link income level to income elasticity of demand (ω_1). Finally we find the link between pollution and income level through the link between pollution and the income elasticity of demand.

PROPOSITION 2. If income elasticity of consumption is negatively correlated with the level of income (concave Engel curve)⁵⁰ then complementarity between goals is more likely the smaller the drop in consumption caused by pollution abatement⁵¹ (through lower price elasticity of consumption), the cleaner the energy (due to more efficient

⁴⁹ See survey by Gahvari and Tsang (2008).

⁵⁰ Empirical evidence pointing towards concavity has been found in Gahvari and Tsang (2008)

⁵¹ There are two stages in the link between pollution abatement and consumption. First reductions in total pollution increase the price of consumption goods. Second the increase in price reduces consumption; this effect depends upon the price elasticity of consumption.

technologies or less carbon intensive sources), and the bigger the effect of pollution abatement on allowance and energy prices.

See proof in Appendix.

Let us recall that $\frac{dc_1}{dp^0} = \frac{dc_1}{dr} \frac{dr}{dp^0}$ and $\frac{dc_2}{dp^0} = \frac{dc_2}{dr} \frac{dr}{dp^0} + \frac{dc_2}{d\varepsilon_2} \left[p_p + p^0 \frac{dp_p}{dp^0} \right]$. Therefore

pollution abatement affects consumption of household group 1 through its effect on consumption good's price (substitution effect). On the other hand it affects consumption of group 2 through both prices and changes in income from revenue transfers

$\left(p_p + p^0 \frac{dp_p}{dp^0} \right)$. Therefore the mechanism underlying the link between pollution and

transfers to low income households unfolds as follows. Under high market effects (increases in prices) of reductions in the cap a positive effect of abatement on income of

household group 2 becomes likely (through $\left(p_p + p^0 \frac{dp_p}{dp^0} \right)$). The increases in prices,

however, do not affect consumption patterns of low income households given the inelastic nature of their demand. Moreover it does not cause a significant reduction in the use of energy by firms because energy is clean (high e_p) and, hence, production is not significantly affected. Therefore consumption must not drop too much while household group 2 was favored by the positive income effects on increases in permits prices. Increases in welfare are, then, achieved by a redistribution of revenue from allowances to low income households. This is because these households display higher income elasticity of consumption making the transfer more effective in maintaining social welfare. This explains the positive link between complementarity between goals.

7. Pareto Efficiency and Income Distribution

We have derived in section 6 a link between pollution and income distribution and, as a result, we have drawn conclusions in terms of conditions for a complementarity (and trade off) between environmental and equity goals. However no direct link between Pareto efficiency and equity in income distribution has been depicted. We now tackle this important issue.

Deciding on the level of the cap (p^0) neglecting issues of allowance allocation in cap and trade systems (i.e. assuming orthogonality between the distribution of revenues and optimal pollution) may result in deviations from the socially optimal pair (p^0, ε_1) depicted by (24). This in turn causes deadweight losses to society that could be prevented if the government followed equation (24). Deadweight losses can be calculated as the difference between maximum social welfare and social welfare obtained when government operates under the assumption of orthogonality (as it has arguably done in previous policy designs). The former is obtained by incorporating firm's behavior (15) and (16), consumers' behavior (18), government's budget equilibrium condition ($\varepsilon_2(p^0) = p_p(p^0)p^0 - \varepsilon_1(p^0)$), and equilibrium prices (20)-(22) into the social welfare function ($V = \eta_1 u^1(x_1, \lambda^{-1} p^0) + \eta_2 u^2(x_2, \lambda^{-1} p^0)$):

$$V^* = \eta_1 u^1\left(x_1\left(r, I_1, \varepsilon_1(p^0)\right), \lambda^{-1} p^0\right) + \eta_2 u^2\left(x_2\left(r, I_2, p_p(p^0)p^0 - \varepsilon_1(p^0)\right), \lambda^{-1} p^0\right) \quad (26)$$

Social welfare under orthogonality is:

$$V^{orth} = \eta_1 u^1\left(x_1\left(r, I_1, \varepsilon_1\right), \lambda^{-1} p^0\right) + \eta_2 u^2\left(x_2\left(r, I_2, p_p(p^0)p^0 - \varepsilon_1\right), \lambda^{-1} p^0\right) \quad (27)$$

The difference between V^* and V^{orth} is that Pareto optimum allocation of allowances implicitly depicted by (24), i.e. $\varepsilon_1(p^0)$, are incorporated into the former but not the latter. Deadweight losses are denoted by the difference between both; $DWL = V^* - V^{orth}$. The welfare impact of pollution abatement (reductions in the cap) while keeping transfers to group 1 constant can be denoted by (see Appendix):

$$\begin{aligned} \frac{d(DWL)}{dp} = & \eta_1 u_{c_1}^1 \left(\frac{dc_1}{dr} \left(\frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_1}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) \\ & + \eta_2 u_{c_2}^2 \left(\frac{dc_2}{dr} \left(\frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_2}{d\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) \frac{d\varepsilon_1}{dp} \right) \end{aligned}$$

Total deadweight losses are determined by the area below $\frac{d(DWL)}{dp}$ between observed cap p^0 and optimal cap p^* . Therefore:

$$DWL = \int_{p^*}^{p^0} \left\{ \left[\eta_1 u_{c_1}^1 \frac{dc_1}{dr} + \eta_2 u_{c_2}^2 \frac{dc_2}{dr} \right] \frac{dr}{d\varepsilon_1} + \left[\eta_1 u_{c_1}^1 \frac{dc_1}{d\varepsilon_1} + \eta_2 u_{c_2}^2 \frac{dc_2}{d\varepsilon_2} \frac{d\varepsilon_2}{d\varepsilon_1} \right] \right\} \frac{d\varepsilon_1}{dp} dp$$

From the necessary conditions for Pareto efficiency we know that $\eta_1 u_{c_1}^1 = \eta_2 u_{c_2}^2 = b$.

Therefore:

$$DWL = b \int_{p^*}^{p^0} \left\{ \left[\frac{dc_1}{dr} + \frac{dc_2}{dr} \right] \frac{dr}{d\varepsilon_1} + \frac{dc_1}{d\varepsilon_1} + \frac{dc_2}{d\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) \right\} \frac{d\varepsilon_1}{dp} dp \quad (28)$$

There is a deadweight loss of assuming orthogonality (i.e. invariance principle of Coase's theorem) whenever $DWL = W^* - W^{orthogonal} > 0$. First note from (28) that violation of the invariance principle from Coase's theorem is a necessary condition for the existence of a link between efficiency and transfers to low income households. This

violation is, as proved in Proposition 1, the most likely situation. We present now an important result linking efficiency and income distribution.

PROPOSITION 3. Efficiency requires greater transfers to low income households (and hence more equitable income distribution) the higher the effect of these transfers on the price of permits.

See proof in Appendix.

The mechanism underlying the result in Proposition 3 is as follows. Transferring revenues to household group 1 has a positive effect on marginal social benefit coming from increases in consumption of this group. The welfare gain is positively linked with income elasticity of demand from group 1 which means that (under concave Engel curves) greater transfers to one group are desirable the lower the group's income.

On the other hand increasing transfers to group 1 implies reducing transfers to group 2 and their consumption. The welfare loss associated with this will be positively linked to their income elasticity of consumption. Moreover increasing transfers to group 1 may affect the equilibrium price of permits which, in turn, will affect total government revenues and transfers to group 2 (through the government's budget equilibrium equation $\varepsilon_2 = p^0 p_p(p^0, \varepsilon_1, I) - \varepsilon_1$). This second effect depends upon the impact of transfers to group 1 on the price of permits. If the impact is negative (increases in transfers to group 1 reduces the price of permits) then group 2 will face a drop in the value of the permits it receives every period and a reduction in consumption. This can only be compensated with transfers to group 2 at the expense of group 1 (low income households).

8. Existence of Efficient Solution

We have derived the link between permit distribution and optimal cap and this link is depicted by equation (24). As mentioned before a wide range of levels of pollution can be rationalized by manipulating the distribution of revenue through lump sum transfers (ε_1). But can *any* level of pollution be rationalized through ε_1 ? If the answer to this question is affirmative then it does not matter how low a pollution level might be, there is an efficient way of implementing it through markets. If the answer is no then this means that implementing reductions of pollution beyond a certain level through markets will entail deadweight losses and these have to be accounted for when evaluating the policy.

We say that a cap p^0 can be rationalized whenever there exists a lump sum transfer to poor households (ε_1) that is both positive and feasible (the lump sum transfer does not exceed total revenue from selling of permits) and such that Pareto optimality (i.e. equation (24)) holds for the pair (p^0, ε_1) . In technical terms any level of pollution can be rationalized whenever:

$$\exists \varepsilon_1 = \varepsilon_1^* : G(p^0, \varepsilon_1^*) = 0 \text{ and } 0 < \varepsilon_1^* < p^0 p_p(p^0, \varepsilon_1^*, \theta), \forall p^0 > 0.$$

The discussion above is illustrated graphically below. We plot in Figure 2 the implicit function $G(p^0, \varepsilon_1^*) = 0$ depicting Pareto efficient combinations.

Any pollution level can be implemented efficiently as long as the intercept of the function (i.e. $\varepsilon_1(0)$) is positive and finite. Functions G^1 and G^2 are examples of economies in which any pollution level (including zero) can be implemented without any deadweight losses. Functions $G^{1'}$ and $G^{2'}$ exemplify economies in which there are no

positive and finite levels of ε_1 that can be implement very low levels of pollution (including zero) without deadweight losses.

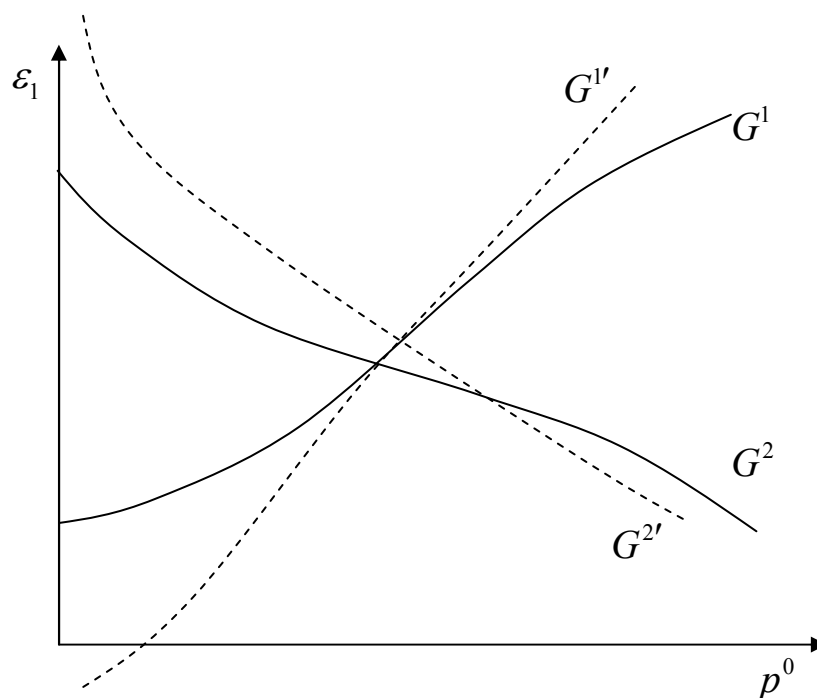


Figure 2. Efficient Permit Distributions

Of course existence of a positive finite intercept will depend upon topological properties of $G(p^0, \varepsilon_1^*)$ which in turn depend upon properties of preferences and technology. Note, in addition, that estimation of $G(p^0, \varepsilon_1^*)$ requires knowledge of demand corresponding to each income group and technology. Empirical information used by computable general equilibrium models of the US economy may shed some light into this implicit function and its asymptotic behavior. This can in turn be useful in understanding and quantifying the welfare losses involved in implementing the cap through markets.

9. Conclusions

The present study analyses the validity of assuming independence between allocation of allowances (or transfer of revenue from permit selling) and Pareto efficiency in the context of cap and trade systems and its implications in terms of convenient ways to allocate allowances or revenue from the selling of those allowances. We find in this study that the link permit distribution and Pareto efficiency depends upon the relationship between allowance allocation to low income households and optimal aggregate level of pollution. If there is no link between these two (i.e. the invariance principle of Coase's theorem holds) then there is no relationship between transfers to low income households and efficiency. However, this study finds (Proposition 1) that invariance holds under very implausible configurations of preferences and technology. Therefore Coase's theorem may have been inappropriately invoked in discussions on how to allocate allowances given the exogenously established cap which may in turn result in welfare losses.

Regarding the link between allowance allocation and optimal pollution (i.e. the invariance principle does not hold) this study finds (Proposition 2) that pollution abatement would require higher transfers to low income households the smaller the drop in consumption caused by pollution abatement (through lower price elasticity of consumption), the cleaner the energy (due to more efficient technologies or less carbon intensive sources), and the bigger the effect of pollution abatement on equilibrium prices of allowances and energy (Proposition 2).

If a relationship between allowance allocation and optimal pollution exists then changes in the allocation while keeping the cap constant yield welfare losses (Pareto inefficiency).

This study has derived a theoretical relationship between transfers to low income households (LIHH) and Pareto efficiency. In particular we find (Proposition 3) that efficiency requires greater transfers to low income households (and hence more equitable income distribution) the higher the effect of these transfers on the price of permits. The effect of transfers on the price of permits is important because it affects the value of permits held by high income households (HIHH). If transfers to LIHH increase the price of permits then the value of allowances received by HIHH every period is increased which increases their income and consumption. This helps alleviate welfare losses of HIHH caused by income redistribution. Therefore, in this case, achievement of Pareto efficiency through cap and trade may require a more equitable income distribution (efficiency and equity are compatible goals).

Finally this study finds (as depicted by implicit function $G(p^0, \varepsilon_1^*)$ in equation (24) and illustrated in Figure 2) that a wide range of caps can be rationalized (or efficiently implemented) through a market economy by manipulating the allocation of allowances. This is a relevant result due to the fact that caps will probably be determined not based on an economic criterion but rather a scientific one (presumably incorporating irreversibility issues). In this context it is our task as economists to find the allowance allocation that maximizes welfare while implementing this cap. Pareto efficiency however is not guaranteed for any level of the cap. In fact there may not be an allowance allocation that rationalizes a given cap and, in such a case, the implementation of the cap entails welfare losses. But even if no Pareto efficient allocation exists, an understanding and quantification of the welfare losses involved in implementing the cap through markets is of great interest.

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Appendix

Proof of compactness and convexity of \mathfrak{Z} .

Using equations of motion (2) and (3) we may define the set of admissible paths as:

$$\mathfrak{S} = \left\{ \begin{array}{l} (k_t, k_{t+1}) \in \mathbb{R}_+^2 : (1-\mu)k_t \leq k_{t+1} \leq f_t(k_t, e_t(p_t)) + (1-\mu)k_t \\ (P_t, P_{t+1}) \in \mathbb{R}_+^2 : (1-\lambda)P_t \leq P_{t+1} \leq f_t(k_t, e_t(\lambda^{-1}P_t)) + (1-\lambda)P_t \end{array} \right\}$$

Output $f_t(k_t, e_t(\lambda^{-1}P_t))$ is positive and then $f_t(k_t, e_t(p_t)) + (1-\mu)k_t > (1-\mu)k_t$.

Therefore compactness of both sets implies convexity. Since k_{t+1} and P_{t+1} belong to \mathbb{R}_+ then both sets are compact.

Proof of Proposition 1.

The invariance principle holds if and only if:

$$\frac{\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\} + \left\{ \gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi \right\}}{-\left\{ \gamma \left[\frac{dc_2}{d\varepsilon_1} \right] \right\} + \left\{ \kappa \left[\frac{dc_1}{d\varepsilon_1} \right] + \alpha \left[\frac{dc_2}{d\varepsilon_1} \right] - \psi \right\}} = 0. \text{ This condition holds if and only}$$

if the numerator is zero or the denominator is infinite. The former is true whenever

$$\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\} = -\left\{ \gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi \right\}, \text{ or } \frac{dr}{dp^0} = \tau = \phi = \chi = 0. \text{ In turn}$$

from the expressions for $\frac{dr}{dp^0}$, τ , ϕ , and χ we know that these are zero if and only if

preferences are linear in pollution (i.e. $u_{c_1 p} = 0$ which results in $\tau = 0$), preferences are additively separable in consumption and pollution (i.e. $u_{c_1 p} = 0$ which results in $\gamma = \phi = 0$), no market effects are caused by changes in the cap (i.e.

$\frac{dr}{d\varepsilon_1} = \frac{dp_p}{d\varepsilon_1} = \frac{dp_E}{d\varepsilon_1} = 0$), and a linear relationship between energy and pollution exists (i.e.

$e_{pp} = 0$ which jointly result in $\chi = 0$). The denominator will be infinite whenever $\omega_1 \rightarrow$

$-\infty$, $\omega_2 \rightarrow -\infty$, or market effects are infinite (i.e. $\frac{dr}{d\varepsilon_1} = \frac{dp_p}{d\varepsilon_1} = \frac{dp_E}{d\varepsilon_1} = \infty$) ■

Proof of Proposition 2.

If income elasticity of consumption is negatively correlated with the level of income (concave Engel curve) and household group 1 earns a very low level of income then income elasticity of consumption of household group 1 will be very high. This will likely

make the denominator negative; $G_{\varepsilon_1}(p^0, \varepsilon_1) < 0$. Then $\frac{dp^0}{d\varepsilon_1} < 0$ if and only if

$G_{p^0}(p^0, \varepsilon_1) > 0$. In turn $G_{p^0}(p^0, \varepsilon_1) > 0$ if and only if

$$\left\{ \left[-\alpha \frac{dc_2}{dr} - \beta \frac{dc_1}{dr} \right] \frac{dr}{dp^0} - \tau \right\} + \left\{ \gamma \frac{dc_2}{dr} \frac{dr}{dp^0} + \phi + \chi \right\} > 0. \text{ Given that } \alpha < 0, \beta < 0, \gamma > 0,$$

$\phi > 0$, $\kappa < 0$, and $\tau < 0$, then the latter result is more likely whenever $\frac{dc_1}{dr} \rightarrow 0$,

$$\frac{dc_2}{dr} \rightarrow 0, \frac{dp_p}{dp^0} \rightarrow -\infty, \frac{dp_p}{dp^0} \rightarrow -\infty, \frac{dp_E}{dp^0} \rightarrow -\infty, \text{ and } e_p \rightarrow \infty. \blacksquare$$

Proof of Proposition 3.

Provided $\frac{d\varepsilon_1}{dp} \neq 0$ holds, then the only way for the social planner to achieve zero

deadweight loss is to set the pollution cap and revenue distribution in such a way that

$$\left\{ \left[\frac{dc_1}{d\varepsilon_1} + \frac{dc_1}{dr} \frac{dr}{d\varepsilon_1} \right] + \left[-\frac{dc_2}{d\varepsilon_2} + \frac{dc_2}{d\varepsilon_2} \frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} + \frac{dc_2}{dr} \frac{dr}{d\varepsilon_1} \right] \right\} = 0. \text{ This in turn means that}$$

achieving Pareto efficiency requires weighing the benefits of transferring income to

group 1 against its cost. Benefits and costs are determined by increases in consumption of group 1 due to income effects ($\frac{dc_1}{d\varepsilon_1}$), reductions in consumption of group 2 due to negative income effects ($-\frac{dc_2}{d\varepsilon_2}$), and changes in consumption of groups 1 and 2 due to changes in prices ($\frac{dc_1}{dr} \frac{dr}{d\varepsilon_1}$, $\frac{dc_2}{dr} \frac{dr}{d\varepsilon_1}$, and $\frac{dc_2}{d\varepsilon_2} \frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1}$).⁵² If group 1 earns a very low level of income relative to group 2 then income elasticity of demand from group 1 will be high relative to that from group 2 rendering:

$$\left[\frac{dc_1}{d\varepsilon_1} + \frac{dc_1}{dr} \frac{dr}{d\varepsilon_1} \right] > \left[\frac{dc_2}{d\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) + \frac{dc_2}{dr} \frac{dr}{d\varepsilon_1} \right].$$

▪ If $\left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) > 0$ equalization of both sides of the inequality requires a reduction in income elasticity of demand from group 1 and an increase in that from group 2. Under concave Engel curves this is accomplished through an increase in transfers to group 1 (low income households) and a reduction in transfers to group 2 (high income households). Therefore, all else constant, increases in transfers to low income households enhance Pareto efficiency (i.e. minimize deadweight losses).

▪ If $\left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) < 0$ while transfers to low income households will reduce income elasticity of demand from group 1 reducing the left hand side of the inequality they will also increase income elasticity of demand from group 2. The latter effect tends to reduce the right hand side of the inequality increasing deadweight losses. Therefore it is not clear

⁵² Note that changes in the price of permits only affect consumption of group 2. This is because we modeled transfers to group 2 from government's budget equilibrium as $\varepsilon_2 = p^0 p_p (p^0, \varepsilon_1, I) - \varepsilon_1$.

in this case that increases in efficiency are positively correlated with transfers to low income households.

- Finally since $\frac{d\varepsilon_2}{dp_p} > 0$ (i.e. increases in the value of permits increases the value of

income transfers to group 2) then $\left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) > 0$ (and a positive link between

efficiency and transfers to low income households) is more likely the higher $\frac{dp_p}{d\varepsilon_1}$ ■

Deadweight Losses from Orthogonality Assumption

$$\begin{aligned} \frac{d(DWL)}{dp} &= V_p^* - V_p^{orth} = \\ &= \overbrace{\eta_1 u_{c_1}^1 \left(\frac{dc_1}{dr} \left(\frac{dr}{dp} + \frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_1}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \eta_2 u_{c_2}^2 \left(\frac{dc_2}{dr} \left(\frac{dr}{dp} + \frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_2}{d\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) \frac{d\varepsilon_1}{dp} \right)}^{MSB^*} \\ &\quad + \overbrace{\eta_1 u_p^1 + \eta_2 u_p^2}^{MSC^*} - \left\{ \overbrace{\eta_1 u_{c_1}^1 \left(\frac{dc_1}{dr} \frac{dr}{dp} \right) + \eta_2 u_{c_2}^2 \left(\frac{dc_2}{dr} \frac{dr}{dp} \right)}^{MSB^0} + \overbrace{\eta_1 u_p^1 + \eta_2 u_p^2}^{MSC^0} \right\} \end{aligned}$$

Cancellation of corresponding terms yields:

$$\frac{d(DWL)}{dp} = \eta_1 u_{c_1}^1 \left(\frac{dc_1}{dr} \left(\frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_1}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \eta_2 u_{c_2}^2 \left(\frac{dc_2}{dr} \left(\frac{dr}{d\varepsilon_1} \frac{d\varepsilon_1}{dp} \right) + \frac{dc_2}{d\varepsilon_2} \left(\frac{d\varepsilon_2}{dp_p} \frac{dp_p}{d\varepsilon_1} - 1 \right) \frac{d\varepsilon_1}{dp} \right) \quad \blacksquare$$